

multiplications at most; with our extremely sluggish computer, that's 66 seconds, roughly one minute!

#37: The determinant is congruent ~~to~~ (mod 5) to:

$$\begin{vmatrix} 1 & & & & & & \\ 0 & 2 & & * & & & \\ 0 & 0 & 3 & & & & \\ 0 & 0 & 0 & 1 & & & \\ 0 & 0 & 0 & 0 & 0 & 3 & \\ 0 & 0 & 0 & 0 & 3 & 0 & \end{vmatrix} \begin{array}{l} \text{exchange} \\ \dots \\ \end{array} \begin{array}{l} \equiv (-1) \cdot 1 \cdot 2 \cdot 3 \cdot 1 \cdot 3 \cdot 3 \equiv 54 \\ \equiv 4 \pmod{5} \end{array}$$

hence not 0.

The \* stands for all the other entries that don't affect the result.

#38: The question amounts to asking for the largest  $n$  such that  $10^n \mid 93!$ . We need to count prime factors 2 and 5 in

$$93! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot \dots \cdot 92 \cdot 93$$

(single) factors 5 come from 5, 10, 15, ... 90; that's  $\frac{90}{5} = 18$

But in 25, 50, 75, we undercounted the factors 5: We only counted one, but should have counted two each.

So that's 21 factors 5 altogether.

There are more factors 2, namely ~~at least~~ more than  $\frac{92}{2} = 46$ .

Hence  $10^{21} \mid 93!$ , but  $10^{22} \nmid 93!$

There are therefore 21 trailing zeros in  $93!$