So even if we could practically restrict the calculation to prime divisors, we would only save a factor ≈ 57 of work and still have $> 10^{21}$ years of work.

#36: (a) $2^n = 10^{n \log_{10} 2} = 10^{n \ln 2/\ln 10} \approx 10^{0.3\,n}$

So if n has 100 digits, $0.3\,n$ has 99 or 100 digits

2^n has $0.3\,n$ digits, and this is a number with (say) 99 digits

So 2^n has roughly 10^{99} digits !!

With one atom of storage medium per digits and $6 \cdot 10^{23}$ atoms per ounce, we'd need $10^{99}/6 \cdot 10^{23} \approx 1.7 \cdot 10^{75}$ ounces of storage medium. With 16 oc per pound thats $\approx 10^{74}$ pounds. With 2000 pounds per ton (a bit more if we're talking metric tons) that's $5 \cdot 10^{71}$ tons. Our train cargo has only 10^4 tons

Even the mass of the earth is only $6 \cdot 10^{21}$ tons!

(b) The exponent of 2 is $n-1$ (not n as stated wrongly in the problem: We want to calculate $2^{n-1} \bmod n$.

The algorithm discussed in class squares 2 repeatedly, reducing mod n whenever feasible, and the exponent $n-1$ (roughly $10^{100} \approx 2^{100/\log_{10} 2} \approx 2^{330}$) gets divided by two in each step. So after 330 multiplications the exponent is $2^0 = 1$

Taking into account the remainders, which still need to be multiplied, this is at most 330 more multiplications.

Altogether the calculation of 2^{n-1} mod n requires 660