Other eqns can be used to the same end:

$x \cdot x = 0$ has <u>two</u> sol's in $\mathbb{Z}_4$ (namely $[0]_4$, $[2]_4$)

but only <u>one</u> sol'n in $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ (namely $([0]_2, [0]_2)$)

$x \cdot x = x$ has <u>two</u> sol's in $\mathbb{Z}_4$ (namely $[0]_4$, $[1]_4$)

but <u>four</u> sol's in $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ (namely all)

$x \cdot x = 1$ has <u>two</u> sol's in $\mathbb{Z}_4$ (namely $[1]_4$, $[3]_4$)

but <u>one</u> sol'n in $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ (namely $([1]_2, [1]_2)$)

<u>34:</u> To the element $A \in \mathcal{P}(M)$, i.e., to a subset $A \subseteq M$, assign the function $f_A : M \to \mathbb{Z}_2$ defined by

$$f_A(x) = \begin{cases} [0]_2 & \text{if } x \notin A \\ [1]_2 & \text{if } x \in A \end{cases}$$

This $\theta : A \mapsto f_A$ is an isomorphism.

<u>Proof:</u> Must show (1) $\theta(A+B) = \theta(A) + \theta(B)$, i.e.

$$f_{(A \setminus B) \cup (B \setminus A)}(x) = f_A(x) + f_B(x) \text{ for all } x \in M$$

(2) $\theta(A \cdot B) = \theta(A) \cdot \theta(B)$, i.e.,

$$f_{A \cap B}(x) = f_A(x) \cdot f_B(x) \text{ for all } x \in M$$

(This will account for "homomorphism")

(3) θ is one-to-one and onto.

<u>Proof of (1), (2)</u>

: 4 cases :	$x \in A, x \in B$	$x \in A, x \notin B$	$x \notin A, x \in B$	$x \notin A, x \notin B$
$f_{(A \setminus B) \cup (B \setminus A)}(x)$	$[0]$	$[1]$	$[1]$	$[0]$
$f_A(x) + f_B(x)$	$[1]+[1]=[0]$	$[1]+[0]=[1]$	$[0]+[1]=[1]$	$[0]+[0]=[0]$
$f_{A \cap B}(x)$	$[1]$	$[0]$	$[0]$	$[0]$
$f_A(x) \cdot f_B(x)$	$[1]\cdot[1]=[1]$	$[1]\cdot[0]=[0]$	$[0]\cdot[1]=[0]$	$[0]\cdot[0]=[0]$