#33:

x	$[0]_{12}$	$[1]_{12}$
$\theta(x)$	$([0]_3, [0]_4)$	$([1]_3, [1]_4)$

let me omit the brackets for the rest...

x	2	3	4	5	6	7	8
$\theta(x)$	(2,2)	(0,3)	(1,0)	(2,1)	(0,2)	(1,3)	(2,0)

x	9	10	11
$\theta(x)$	(0,1)	(1,2)	(2,3)

For $\theta: \mathbb{Z}_{10} \to \mathbb{Z}_2 \oplus \mathbb{Z}_5$, I also omit the brackets after the first sample

x	$[0]_{10}$	1	2	3	4	5	6	7	8	9
$\theta(x)$	$([0]_2, [0]_5)$	(1,1)	(0,2)	(1,3)	(0,4)	(1,0)	(0,1)	(1,2)	(0,3)	(1,4)

It is <u>correctly observed</u>, but <u>not a sufficient answer</u> to the problem

to note that $\theta: \mathbb{Z}_4 \to \mathbb{Z}_2 \oplus \mathbb{Z}_2$

$$\begin{aligned} [0] &\mapsto ([0],[0]) \\ [1] &\mapsto ([1],[1]) \\ [2] &\mapsto ([0],[0]) \\ [3] &\mapsto ([1],[1]) \end{aligned}$$

is not an isomorphism, as it is neither one-to-one nor onto.

We must show that <u>NO</u> mapping $\vartheta: \mathbb{Z}_4 \to \mathbb{Z}_2 \oplus \mathbb{Z}_2$ is an isomorphism.

For <u>any</u> isomorphism $\vartheta: \mathbb{Z}_4 \to \mathbb{Z}_2 \oplus \mathbb{Z}_2$ (if such an isom. existed), it would follow: If x solves $x+x=0$ in $\mathbb{Z}_4$ then $\vartheta(x)$ solves $y+y=0$ in $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ and conversely, if y solves $y+y$ in $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ then $\vartheta^{-1}(y)$ solves $x+x=0$ in $\mathbb{Z}_4$.

The eqn $x+x=0$ must have the <u>same</u> number of solutions in isomorphic rings.

But $x+x=0$ has <u>two</u> sol's in $\mathbb{Z}_4$, namely $[0]_4$, $[2]_4$, but <u>four</u> sol's in $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ (all elements of $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ satisfy this eqn.) So $\mathbb{Z}_4$ and $\mathbb{Z}_2 \oplus \mathbb{Z}_2$ cannot be isomorphic