- The surprise announced in the text of the pblm has already been mentioned in class:

  The prime 3 divides the product $(2+\sqrt{-5})(2-\sqrt{-5}) = 9$, but it does not divide either factor.

  This shows that there cannot be a euclidean algorithm in the ring $\mathbb{Z}[\sqrt{-5}]$. Otherwise we would conclude (as in the proof for $\mathbb{Z}$) that $p | ab \Rightarrow p | a$ or $p | b$

#30: Let $x = 5555^{5555}$ and $Q(Q(Q(x))) = y$

We know about y two things:

(1) $y \equiv x \mod 9$

(2) y can be at most ... (well we'll figure out how much)

- Using (1), we say that

$$5555 \equiv 20 \equiv 2 \mod 9 \text{, hence}$$

$$x = 5555^{5555} \equiv 2^{5555} \equiv 2^{6 \cdot 925 + 5} =$$

$$(2^6)^{925} \cdot 2^5 \equiv 1^{925} \cdot 32 \equiv 5 \mod 9$$

Therefore $y \equiv 5 \mod 9$

- Carrying out (2), we see that $x = 5555^{5555} < 10^{4 \cdot 5555} = 10^{22220}$

Hence x has at most 22220 digits and

$$Q(x) \leq 22220 \cdot 9 = 199980$$

Therefore $Q(x)$ has at most 6 digits, and if it does indeed have exactly 6 digits, then the first digit is 1.