#28: $13 = (3+2i)(3-2i)$

hence 13 is not a prime in the ring $\mathbb{Z}[i]$

#29: • Suppose $(a+b\sqrt{-5})(c+d\sqrt{-5}) = 1$ with $a,b,c,d \in \mathbb{Z}$

Then, by taking the complex conjugate, we also have

$$(a-b\sqrt{-5})(c-d\sqrt{-5}) = 1$$

Multiplying both eqn's, get

$$(a^2+5b^2)(c^2+5d^2) = 1$$

So $a^2+5b^2 = 1$, $c^2+5d^2 = 1$ because both factors are <u>nonneg. integers</u>!

this can only happen if $b=0$, $a=\pm 1$ (again use: $a,b \in \mathbb{Z}$)

this can only happen with $c = \pm 1$, $d=0$.

• Let me give another, more complicated solution, if you do not follow the hint, but try to calculate it in a "pedestrian" way.

$(a+b\sqrt{-5})(c+d\sqrt{-5}) = 1$ means

$$\left.\begin{aligned} ac - 5bd &= 1 \\ bc + ad &= 0 \end{aligned}\right\}$$ two eqn's for four unknowns, but we have <u>extra</u> info: a,b,c,d <u>integers</u>

You can solve for c, d from these eqns:

$$c = \frac{a}{a^2+5b^2} \qquad d = \frac{-b}{a^2+5b^2}$$

For $c \in \mathbb{Z}$, we need either $a=0$, or $|a| \geq a^2+5b^2$

↳ can only happen if $a = \pm 1$, $b=0$