Points, Vectors. and Origins in Linear Algebra

I'd like to clarify a distinction for you that is alas not discussed carefully in the textbook and will therefore cause a bit of confusion when you mx linear algebra and multi-variable calculus. Namely the distinction between point and vector.

GEOMETRY:

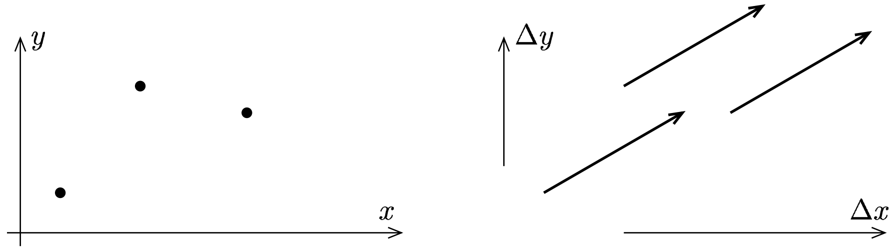
Geometrically, you wouldn't even think of confusing the two. The figure below on the left shows points, the one on the right shows vectors. But notice that the left figure shows *three different points*, whereas the right figure shows *three times the same vector*. This is because you may freely translate a vector (without turning it), but when you translate a point, it becomes a different point.

VIA COORDINATES INTO ALGEBRA:

When you convert these geometric pictures into algebra, you use coordinates. A good name for coordinates of a point is $\begin{bmatrix} x \\ y \end{bmatrix}$, whereas a good name for coordinates of a geometric vector would be $\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$. The Δ in the names Δx and Δy reminds us of the word 'difference', because a vector is now very much like a difference between the end point and the start point: $\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} = \begin{bmatrix} x_{\text{end}} \\ y_{\text{end}} \end{bmatrix} - \begin{bmatrix} x_{\text{start}} \\ y_{\text{start}} \end{bmatrix}$.

If we don't focus on the distinction between points and vectors (but only deal with vectors alone), we may want an easy notation and will call coordinates for a vector $\begin{bmatrix} x \\ y \end{bmatrix}$, rather than $\begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$. But this is not meant to deny the obvious geometric distinction between points and vectors.

In the figure below, I have drawn coordinate axes. In order to find coordinates for the points, you need to know the origin of the coordinate axes (as well as the unit length and direction for each axis). In contrast, if you want to find coordinates for a vector, you only need the direction and unit length for each coordinate axis, but you do not need the origin. To make this distinction more visible, I have avoided showing the origin in the right figure.



Mix Calculus with Linear Algera:

In multi-variable calculus, it is popular to treat points as vectors. This allows the convenient use of matrix algebra techniques and notation in calculus. But it obfuscates the geometric distinction between points and vectors. The way they are doing this is simple, but it *requires an origin* of the coordinate system. They will identify a point with the vector that goes from the origin to the point in question.

When they do this, they always think of vectors as anchored in the origin, and they don't care to translate vectors around.

When we discuss the connection between matrices and linear transformations of the plane, we will also borrow this calculus point of view in order to give nice geometric pictures that illustrate the effect of a matrix. If we were to plot 20 or so vectors, we would easily get an unintuitive, clogged figure, but with 20 or so points, we can draw nice and lucid figures.

The next figure illustrates how points are identified with vectors by means of an origin.