Hwk #6:

I love to collect airline miles. Let's assume I fly from Atlanta (ATL) to Frankfurt, Germany (FRA). How many miles is the shortest distance? I look up the following info on a map: ATL is at 84.4° western longitude and 33.65° northern latitude. FRA is at 8.6° eastern longitude and 50.1° northern latitude. The radius of the earth is 3975 mi.

Transforming from spherical to cartesian coordinates (the equator plane is the xy plane, with the Greenwich meridian going through the x axis); and using the dot product again, I can calculate the number of miles for this trip (along the shortest route, which is the arc of a circle centered at the center of the earth and connecting from ATL to FRA).

(Note: It is of course more expedient to use symbols $\lambda_{1,2}$ and $\varphi_{1,2}$ for the coordinates first, and postpone plugging in numbers until the end.)

Hwk #7:

Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by $f(x,y) = \frac{xy}{x^2+y^2}$ for $(x,y) \neq (0,0)$ and $f(0,0) = 0$. By using polar cordinates, draw the level curves of this function. As explained in class, this function is *not* continuous at $(0,0)$. Show that nevertheless, all the single variable functions g and h given by $g(x) := f(x,y)$ for any choice of y, and $h(y) := f(x,y)$ for any choice of x, are continuous. For any fixed k, find the limit $\lim_{x\to 0} f(x,kx)$.

Hwk #8:

Cranking the previous example up a notch, consider the function f given by $f(x,y) := \frac{x^2y^4}{x^4+y^8}$ for $(x,y) \neq (0,0)$ and $f(0,0) = 0$.

Show that each of the radial limits $\lim_{x\to 0} f(x,kx)$ and $\lim_{y\to 0} f(0,y)$ equal $0 = f(0,0)$, but that, nevertheless, $\lim_{(x,y)\to(0,0)} f(x,y)$ does not exist (and hence f is not continuous in the origin). Draw level lines of f to get insight into this function.

Can you describe a 'curve of approach' in the (x,y) plane along which the single variable limit exists, but is different from 0? That is, can you find $x(t)$ and $y(t)$ such that $\lim_{t\to\infty} x(t) = 0$ and $\lim_{t\to\infty} y(t) = 0$, but $\lim_{t\to\infty} f(x(t),y(t)) \neq 0$.

Can you also describe a curve of approach for which the limit does not exist?

Hwk #9:

Draw level curves of the function f given by $f(x,y) := |x| + |y|$, and describe the graph $z = f(x,y)$.

Same question for $g(x,y) = \sqrt{x^2+y^2}$.

Hwk #10:

Does $\lim_{(x,y)\to(0,0)} y \sin\frac{1}{x}$ exist, and if so, what is its value? Explain.