Hwk #29:

(For warmup only: this is predominantly single variable calculus)
Consider $f(x) := \int_0^\infty e^{-xt}\frac{\sin t}{t}\,dt$. You will not find an immediate antiderivative by which to evaluate this integral. Nevertheless, calculate $f'(x)$. [The Math 447 expert tells you that moving the x-derivative under the t-integral is legitimate here.] You should be able to evaluate the integral that you obtain for $f'(x)$.

What do you think $\lim_{x\to\infty} f(x)$ is? [The Math 447 expert tells you that in this example it is legitimate to move $\lim_{x\to\infty}$ past the integral sign.]

Finally, knowing $f'(x)$, $\lim_{x\to\infty} f(x)$ and the fundamental theorem of calculus, find $f(x)$. Specifically, what is $\int_0^\infty \frac{\sin t}{t}\,dt$?

The late physics Nobel prize winner Richard Feynman reports in his memoirs how, as a student, he got the reputation of being an integration wizard, because he was familiar with this 'differentiation under the integral sign technique', which his peers hadn't learned.

Hwk #30:

Use the multi-variable chain rule to determine $f'(x)$, when $f(x) := \int_0^x \frac{\sin(xt)}{t}\,t$.

Analogous question for $g(x) := \int_{x/2}^{2x} \frac{e^{xt}}{t}\,dt$.

Again, we rely on the Math 447 expert, who tells us that it is legitimate to move derivatives past the integral sign in this example.

Hwk #31:

A quantity w depends on the coordinates x, y, z in 3-space as follows: $w = x^2 + y^2 + xyz$ (1). We study w especially on the plane given by $z = x + 2y$. Then we have there $w = x^2 + y^2 + xy(x + 2y) = x^2 + y^2 + x^2y + 2xy^2$ (2).

Now we calculate $\frac{\partial w}{\partial x}$ from (1): $\frac{\partial w}{\partial x} = 2x + yz$. On the plane, this simplifies to $\frac{\partial w}{\partial x} = 2x + y(x + 2y) = 2x + xy + 2y^2$.

Calculating $\frac{\partial w}{\partial x}$ on the plane directly from (2), we get $\frac{\partial w}{\partial x} = 2x + y(x+2y) = 2x + 2xy + 2y^2$. We clearly have a discrepancy by a term xy. What is wrong? Clear up the confusion. (This requires some text as well as formulas.)