Hwk #21:

Show that for $f(\vec{x}) := \|\vec{x}\|$ we have $\nabla f(\vec{x}) = \vec{x}/\|\vec{x}\|$ at $\vec{x} \neq \vec{0}$.

Hwk #22:

Consider the vector $\vec{e}$ and any (non-empty) level set of the function $g(\vec{x}) := \|\vec{x}-\vec{e}\|+\|\vec{x}+\vec{e}\|$. We know from #13 that the level set is an ellipse. In this problem we show the reflection property of the ellipse: A ray emanating from one focus $\vec{e}$ and reflected in the ellipse will pass through the other focus $-\vec{e}$. The reflection law in physics says that the incoming ray has the same angle with the normal to a curve as the reflected ray.

Prove the reflection property of the ellipse by checking the angle between the normal to the ellipse and the ray or reflected ray respectively. You can do everything without coordinates or components, just using vector notation.

Hwk #23:

Use the result of Hwk #15 together with the multi-variable chain rule to obtain a 'power rule' for single variable derivatives, namely $\frac{d}{dx}(f(x)^{g(x)}) =?$.

Obtain the same result by single variable methods alone, writing f^g as $e^{g \ln f}$.

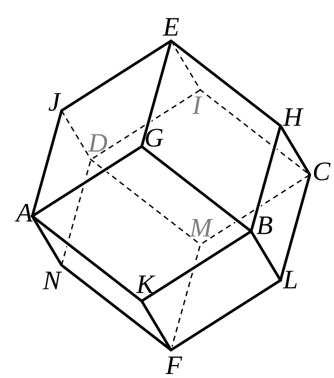
Hwk #24:

Suppose that a piece of a level curve $g(x, y) = c$, near some point (x_0, y_0), where c is some constant, can be written as the graph of a function h: $y = h(x)$. Express the slope $h'(x_0)$ of this graph in terms of partial derivatives of g. Write your result both in mathematical notation and in physicists' notation with differential quotients (with z for the output variable of g and dy/dx expressed in terms of $\partial z/\partial x$ and $\partial z/\partial y$). *if you think a certain minus sign in your result looks weird, you're right: it does look weird, but it's still correct! Or rather I hope so, I haven't seen your solution; all I say is that the correct solution may have some weird looking detail.*

Hwk #25:

A rhombus is a quadrangle whose sides are all of the same length. A rhombododekahedron is a polyhedron with 12 faces, all of which are congruent rhombi. At each vertex, either four rhombi meet with their acute angles, or else, three rhombi meet with their obtuse angles. See the figure.

This does not work with an arbitrary rhombus. If you make the acute angle smaller (thus making the obtuse angle larger), the 'crown' above the zigzag $AGBHCIDJA$ becomes skinnier and taller, and the obtuse angle of the rhombus will become too large to fit below the crown as angle AGB.

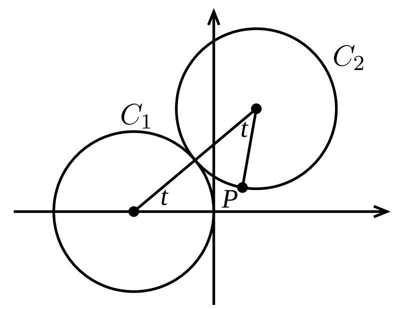


Your job is to find the correct angle for a rhombus that is fit to build a rhombododekahedron. (No calculus here; just training your vector geometry and spatial vision a bit more.)

Hwk #26:

The cardioid is most easily described in terms of polar coordinates: $r = 2(1 - \cos\varphi)$. Its name comes from the Greek word for 'heart', and you'll see why when you graph this curve. So graph it carefully, but don't just steal the graph from a Valentine's card, that would be too corn(er)y!

A circle C_1 of radius 1 sits stationary with center $(-1, 0)$. Another circle C_2 of radius 1 touches it from the right, in the origin. This circle is soon to roll along the fixed circle C_1 without sliding. A point P is marked on the circle C_2. Initially it is the point where both circles touch. As C_2 rolls along C_1, the point P traces out a curve in the stationary plane. Use t for the angle (measured on C_1) of the point of contact of both circles. Give a vector valued function $t \mapsto \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ that describes the position of P as a function of t.



Show that the curve traced out by P is a cardioid.

Hwk #27:

As a point P moves along a curve $r = f(\varphi)$, the line from the origin to P sweeps out a sector-like area. Approximating this area by many small sectors of a circle (a Riemann sum), find an integral formula for this area. Then use it specifically for $f(\varphi) = 2(1 - \cos\varphi)$ to find the area of the cardioid from the previous problem.

Hwk #28:

Given a curve described in parametric form $t \mapsto \vec{x}(t)$, in the plane or in space, we may pretend that the parameter t represents a time and that $\vec{x}(t)$ is the position vector at 'time' t. Then the velocity is $\vec{x}'(t)$, and the speed is $\|\vec{x}'(t)\|$. The length of the curve between parameters t_0 and t_1 is $\int_{t_0}^{t_1} \|\vec{x}'(t)\|\, dt$.

Using this insight, calculate the length (perimeter) of the caridioid.