Hwk #15:

Find $Df(x,y)$ for $f(x,y) = x^y$, where $x > 0$.

Hwk #16:

Consider the following vector valued multi-variable functions:

$$f : (r, \varphi) \mapsto \begin{bmatrix} r\cos\varphi \\ r\sin\varphi \end{bmatrix}$$

defined for $\{(r,\varphi) \mid r > 0\,,\ \varphi \in \mathbb{R}\}$; and

$$g : (r, \vartheta, \phi) \mapsto \begin{bmatrix} r\sin\vartheta\ \cos\phi \\ r\sin\vartheta\ \sin\phi \\ r\cos\vartheta \end{bmatrix}$$

defined for $r > 0$, $\vartheta \in\ ]0, \pi[$, $\phi \in \mathbb{R}$.

Find $Df(t,\varphi)$ and $Dg(r,\vartheta,\phi)$ by calculating the necessary partial derivatives and observing they are continuous.

Hwk #17:

Show that for any collection of real numbers $a_1, a_2, \ldots, a_n$, the following inequality is true:

$$(a_1 + a_2 + \ldots + a_n)^2 \le n(a_1^2 + a_2^2 + \ldots + a_n^2)$$

Hint: use the Cauchy Schwarz inequality on the vector $\vec{a} = [a_1, a_2, \ldots, a_n]^T$ *and another vector* $\vec{b}$ *which you invent conveniently for the purpose.*

Hwk #18:

In this problem you are to check that a certain function is differentiable *by relying on the original definition* rather than using the theorem that makes continuity of partial derivatives sufficient to guarantee differentiability.

Show that $f : (x,y,z) \mapsto \sqrt{x^2 + 2y^2 + z}$ is differentiable at $(1,1,1)$. (No typo; it's z, not z^2, just for variety's sake.)

Strategy: as in the example in class, you use partials to find the only candidate for $Df(1,1,1)$ *first; then you check the limit from the definition. — Remember: when you have a difference involving a square root in a limit problem* ($\sqrt{a} - b$ *small because* a *is close to* b^2), *you write* $\sqrt{a} - b = (a - b^2)/(\sqrt{a} + b)$. *— Another trick that may come in handy is to use the inequality in the previous problem near the end of the calculation.*

Hwk #19:

Assuming f differentiable, show that $\frac{d}{dt} f(\vec{x} + t\vec{v})|_{t=0} = Df(\vec{x})\vec{v}$. (The quantity on theft hand side, if it exists, is called directional derivative of f at $\vec{x}$ in direction $\vec{v}$.

Hwk #20:

The multi-variable chain rule says: $D(f \circ g)(\vec{p}) = Df(g(\vec{p}))Dg(\vec{p})$. Here is one specific example for which I ask you to caclulate all quantities involved in this equation and check the equality, all by explicit calculation.

$$f(x, y, z) := xyz^2 + (y - x)/(1 + z^2), \quad g(\vartheta, \phi) = \begin{bmatrix} \sin\vartheta \cos\phi \\ \sin\vartheta \sin\phi \\ \cos\vartheta \end{bmatrix}.$$

(Motivation for this example: think of $f \circ g$ as a function on the unit sphere.)