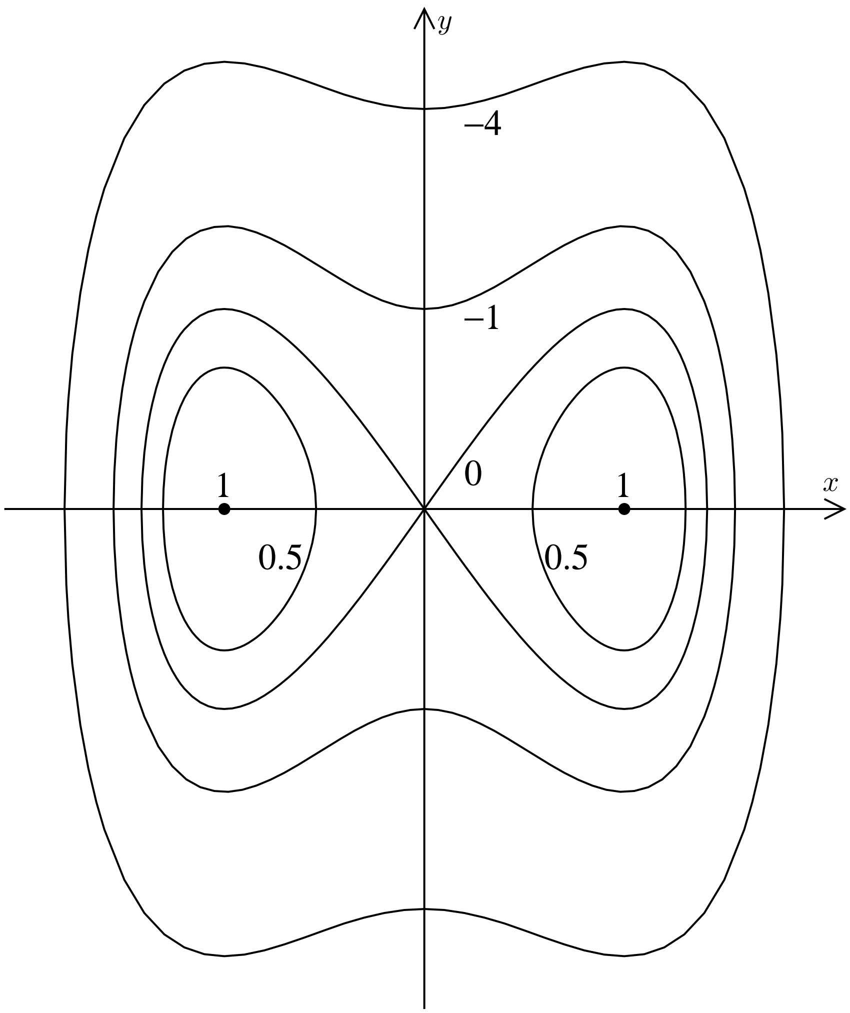
**Hwk #11:**

Try to understand the level sets of the function $f$ given by $f(x,y) = 2x^2 - x^4 - y^2$. In particular use single-variable calculus and simple algebraic reasoning to find maxima of $f$. Make sure to sketch at least five level sets $\{(x,y) \mid f(x,y) = c\}$. Namely, for $c \in \{1, \frac{1}{2}, 0, -1, -4\}$. You may also find it useful to sketch graphs of a few single variable functions $g(x) := f(x, y_0)$ for various $y_0$. Try to avoid using technology that does 'multi-variable graphs', but feel free to enlist the help of technology for single variable graphs if this helps. The skill you are to train here is to piece single variable info together to get a multi-variable picture.

**Solution:** 7 pts

As far as the maximum is concerned, $f(x,y) \le f(x,0) \le f(\pm 1, 0) = 1$. Here, the single variable maximum of $g(x) := f(x,0) = 2x^2 - x^4$ can be found by studying the derivative $g'(x) = 4(x - x^3)$ as usual; or else we can argue that $2x^2 = 2\sqrt{1 \cdot x^4} \le 1 + x^4$ by the agm inequality.




The level set for level 1 consists of only the two points $(\pm 1, 0)$. The level set for level $z$ consists of two sv function graphs $y = \pm\sqrt{-z + 2x^2 - x^4}$. For $0 < z < 1$, the term under the root is non-negative on two intervals, namely for $x^2 \in [1 - \sqrt{1-z}, 1 + \sqrt{1-z}]$. These level sets are in the shape of two (slightly deformed) circles, each surrounding a maximum. For $z = 0$, the two circles merge into a figure-8 curve; for $z < 0$, the term under te square root is non-negative on a single interval centered at 0, and the level curves consist of a single closed curve.

Regarding the graph of $f$, the origin looks like a *saddle*. Looking only into the $x$ direction, it looks like a minimum, but looking into the $y$ direction, it looks like a maximum.

This example displays a general feature: In the level line picture, typical saddles show up as crossings, isolated relative maxima (and also minima) as single point level sets.