Hwk #11:

Try to understand the level sets of the function f given by $f(x,y) = 2x^2 - x^4 - y^2$. In particular use single-variable calculus and simple algebraic reasoning to find maxima of f. Make sure to sketch at least five level sets $\{(x,y) \mid f(x,y) = c\}$. Namely, for $c \in \{1, \frac{1}{2}, 0, -1, -4\}$. You may also find it useful to sketch graphs of a few single variable functions $g(x) := f(x, y_0)$ for various y_0. Try to avoid using technology that does 'multi-variable graphs', but feel free to enlist the help of technology for single variable graphs if this helps. The skill you are to train here is to piece single variable info together to get a multi-variable picture.

Hwk #12:

More about the ellipse: Given the points $F_\pm = (\pm e, 0)$ in the plane (where e is some positive real number, not to be confused with the Euler number $2.718\ldots$), and a number $a > e$. Show that the set of those points $P = (x,y)$ in the plane that satisfy the condition $\|\vec{PF_+}\| + \|\vec{PF_-}\| = 2a$ is an ellipse $x^2/a^2 + y^2/b^2 = 1$. How does b relate to e and a? What is the eccentricity ε of the ellipse?

Hwk #13:

Reconsider the function f from Problem #8: $f(x,y) := \frac{x^2 y^4}{x^4 + y^8}$ for $(x,y) \neq (0,0)$ and $f(0,0) = 0$. Write it in polar coordinates: $g(r,\varphi) := f(r\cos\varphi, r\sin\varphi)$. The partial derivative $\partial g(r,\varphi)/\partial r$ at $r = 0$ is a directional derivative of f (at the origin). Show that all directional derivatives at the origin vanish, so the graph has a horizontal tangent in each direction. Nevertheless, f is not even continuous at the origin.

Plot, for some choice of fixed φ (other than an integer multiple of $\pi/2$) the graph of the single variable function $g(\cdot,\varphi) : r \mapsto g(r,\varphi)$. Include information about the precise location of the maximum of this function.

Hwk #14:

Sketch level lines for the function $f(x,y) := x^3 - 3xy^2$. Choose levels 4, 1, 0, −1, −4. The most convenient way to do this is to use polar coordinates again. Look for a trig formula involving multiple angles that fits the situation (you'd likely not have memorized this formula to recognize it at first sight, that's why I say you should look for it).

This function is hand-picked to display a rare pattern in the level lines picture

Describe the graph of f in topographer's terms: where are the hills and the valleys? The point $(0,0)$ is said to feature a *monkey saddle* of this function f.