

Rational expressions in $\sin u$ and $\cos u$ can always be handled by the substitution $t = \tan \frac{u}{2}$ (sometimes simpler sub's may apply). To carry this out in practice, we need to express $\sin u$ and $\cos u$ in terms of $t = \tan \frac{u}{2}$, which requires juggling some trig identities

$$\tan \frac{u}{2} = t$$

$$1 + \tan^2 \frac{u}{2} = 1 + t^2$$

$$= 1 / \cos^2 \frac{u}{2}$$

from tan to cos
via Pythagoras

$$\text{So } \cos^2 \frac{u}{2} = \frac{1}{1+t^2}$$

use double angle
formula

$$\cos u = 2 \cos^2 \frac{u}{2} - 1 = \frac{2}{1+t^2} - 1 = \frac{1-t^2}{1+t^2}$$

Result:

$$\cos u = \frac{1-t^2}{1+t^2}$$

Then

$$\sin u = 2 \sin \frac{u}{2} \cos \frac{u}{2} = 2 \tan \frac{u}{2} \cdot \cos^2 \frac{u}{2} = 2t \cdot \frac{1}{1+t^2}$$

double angle
formula

Result:

$$\sin u = \frac{2t}{1+t^2}$$

Of course

$$du = \frac{2 dt}{1+t^2}$$

from differentiating $u = 2 \arctan t$