Math 341 Homework 11

1. Use the Cauchy Criterion for convergent sequences of real numbers to supply a proof of Theorem 6.2.5 in the textbook. (First, define a candidate for $f(x)$, then argue that $f_n \to f$ uniformly.)

2. Prove that if (f_n) and (g_n) are uniformly convergent sequences, then so is $(f_n + g_n)$.

3. Let (g_n) be a sequence of continuous functions that converges uniformly to g on a compact set K. If $g(x) \neq 0$ on K, show $(1/g_k)$ converges uniformly on K to $1/g$.

4. Let $f_a(x) = \begin{cases} x^a & \text{if } x > 0 \\ 0 & \text{if } x \leq 0. \end{cases}$

   (a) For which values of a is f continuous at zero?

   (b) For which values of a is f differentiable at zero? In this case, is the derivative function continuous?

   (c) For which values of a is f twice-differentable?

5. What subsets of $\mathbb{R}$ have the property that every *closed* cover (i.e., a cover consisting of closed sets) admits a finite subcover?