Math 323 Chapter 9 Supplemental Problem Solutions

1. The height of a hobbits is normally distributed, with unknown mean, and known standard deviation of 2.2 inches.

   Ten randomly selected hobbits have heights of 51, 47, 48, 48, 43, 50, 42, 49, 45 and 46 inches.

   (a) Give a 95% confidence interval for the average height of hobbits.
   Calculate $\hat{\theta} = 46.9$, and look up that $z_{\alpha/2} = 1.96$. The confidence interval has bounds $\hat{\theta} \pm 1.96\frac{2.2}{\sqrt{10}}$, or the interval $(45.54, 48.26)$.

   (b) How many samples would we need to give a 95% confidence interval with length of 2 inches?
   We need $z_{\alpha/2}\frac{2.2}{\sqrt{n}} \leq 1$. Solving for n we get $n \geq (1.96 \cdot 2.2)^2 = 18.59$, so we need at least $n = 19$ samples.

   (c) Suppose we only want an upper bound for the average height. Find a $\hat{\Theta}_n^+$ so that

   $$P(\theta \leq \hat{\Theta}_n^+) \geq 0.95$$

   We want z_α so that $\phi(z_\alpha) = 0.95$. The standard normal table gives $z_{\alpha/2} = 1.645$. The $\hat{\Theta}_n^+ = \hat{\theta} + 1.645\frac{2.2}{\sqrt{10}} = 48.04$.

2. The height of hobbits is normally distributed, with mean θ and standard deviation v, both unknown constants.

   Ten randomly selected hobbits have heights of 51, 47, 48, 48, 43, 50, 42, 49, 45 and 46 inches.

   (a) Calculate $\hat{\theta}$ and $\hat{S}_n^2$.
   $\hat{\theta} = 46.9$ and $\hat{S}_n^2 = 8.5444$.

   (b) Give a 95% confidence interval for the average height of hobbits.
   Since we must use an estimate of the standard deviation, $\hat{\theta}$ has the t-distribution. The confidence interval will be $\hat{\theta} \pm t_{\alpha/2}\frac{\hat{S}_n}{\sqrt{10}}$ or $46.9 \pm 2.262\frac{2.92}{\sqrt{10}}$ or 46.9 ± 2.09. The 2.262 is $t_{\alpha/2}$ for $n - 1 = 9$ degrees of freedom.

   (c) Suppose we only want an upper bound for the average height. Find a $\hat{\Theta}_n^+$ so that

   $$P(\theta \leq \hat{\Theta}_n^+) = 0.95$$

   using the t-distribution.
   Same idea as the previous problem, but with t. $\hat{\Theta}_n^+ = \hat{\theta} + t_\alpha\frac{\hat{S}_n}{\sqrt{10}} = 46.9 + 1.833\frac{2.92}{\sqrt{10}} = 48.59$

3. The costs of repairing minivan bumpers damaged by a 5-mph collision, for seven models of minivan, are given by 1154, 1106, 1560, 1769, 2299, 1741 and 3179 (dollars). Assume that repair costs are normally distributed.

(a) Calculate $\hat{\theta}$ and $\hat{S}_n^2$.

$\hat{\theta} = 1829.7$ and $\hat{S}_n^2 = 517576$.

(b) Find a 95% confidence interval for the true average repair cost.

Again, a t distribution, this time with 6 degrees of freedom.

$\hat{\theta} \pm t_{\alpha/2}\frac{\hat{S}_n}{\sqrt{7}} = 1829.7 \pm 2.447\frac{719.4}{\sqrt{7}} = 1829.7 \pm 665.4$.

4. A medical researcher wants to determine whether a new treatment will produce beneficial results in a higher proportion of patients than the 0.65 receiving beneficial results from standard treatment. How should the research interpret an experiment if 145 of 200 patients obtained beneficial results with the new treatment? (Use $\alpha = 0.05$.)

   Use $H_0 : p = 0.65$ with $H_1 : p > 0.65$, so a one-sided confidence interval. We'll reject H_0 if the proportion is less than $0.65 + z_\alpha\sqrt{\frac{0.65(1-0.65)}{200}} = 0.65 + 1.645\sqrt{\frac{(0.65)(0.35)}{200}} = 0.705$. Since $145/200 = 0.725$, we reject H_0 and conclude that the new treatment benefits a higher proportion of patients.

5. A random sample of size 100 is drawn from a large lot of manufactured items. We wish to test (at the 0.05 level of significance) whether the proportion of acceptable items in the lot is 0.80, against the alternative that it is less than 0.80.

   (a) What are H_0 and H_1?

   $H_0 : p = 0.80$; $H_1 : p < 0.80$.

   (b) What is the largest number of acceptable items in the sample that will lead to a rejection of the null hypothesis?

   Similarly to the previous problem, we reject H_0 if the sample proportion is less than $0.80 - z_\alpha\sqrt{\frac{(0.8)(0.2)}{100}} = 0.80 - 1.645(0.04) = 0.7342$. So 73 or fewer acceptable items would lead to rejection of H_0.

   (c) Find the probability of accepting H_0 if the actual proportion of items is 0.75.

   We need to compute the probability that a binomial random variable with $n = 100$ and $p = 0.75$ would be 73 or less. This is 0.3583 using the first Google result for "binomial calculator", or 0.365 using the normal approximation.

6. Officials have conjectured that 3 out of 10 cars would fail emissions requirements if emission testing were implemented. In a pilot program, 73 of 400 randomly selected vehicles failed to meet the requirements. Using this data, test the officials' claim at the 0.01 level of significance.

   Again this seems like a one-sided test, with $H_0 : p = 0.3$ and $H_1 : p < 0.3$. We'll reject H_0 if the proportion is less than $0.3 - z_\alpha\sqrt{\frac{(0.3)(0.7)}{400}} = 0.3 - 2.325 \cdot 0.023 = 0.247$. Since 73/400=0.1825 we reject H_0 and conclude that the proportion of vehicles that would not meet the emission requirements is less than 3 in 10.