Math 323 Chapter 3 Problems

1. Find the PDF, mean and variance of a random variable X having CDF

$$F_X(x) = \begin{cases} 1 - \frac{a^2}{x^3} & \text{if } x \geq a \\ 0 & \text{otherwise.} \end{cases}$$

2. The median of a random variable X is the number μ satisfying $F_X(\mu) = \frac{1}{2}$. Find the median of an exponential random variable with parameter λ.

3. X is a random variable with PDF

$$f_X(x) = \begin{cases} 2x/3 & \text{if } 1 \leq x \leq 2 \\ 0 & \text{otherwise,} \end{cases}$$

   and A is the event $A = \{X \geq 1.5\}$. Compute $\mathbb{E}X$, $P(A)$ and $\mathbb{E}[X|A]$.

4. Alice and Bob need to be at the airport in 20 minutes. There are two taxi companies available. One is Exponential Taxis, whose cab arrival times are exponentially distributed with mean 5 minutes. The other is Uniform Cab Company, whose arrival times are uniformly distributed between 0 and 10 minutes. Suppose that Alice calls Exponential Taxis and Bob calls Uniform Cab.

   (a) What is the probability that Alice's cab arrives before Bob's?

   (b) What is the expected time of arrival of the first cab?

   The travel times for both companies are both uniform (5,15). The total time for the cab ride will be the arrival time plus the travel time.

   (c) What are the mean and variance for Alice's cab ride?

   (d) What are the mean and variance for Bob's cab ride?

   (e) What is the probability that Alice's cab will be at the airport in less than 20 minutes?

   (f) What is the probability that Bob's cab will be at the airport in less than 20 minutes?

   (g) If the two travel separately, what is the probability that at least one of them will arrive at the airport too late?

   Suppose instead that Alice and Bob decide to share whichever taxi arrives first.

   (h) What is the expected duration of the shared trip?