

## Hypergeometric sums and combinatorial number theory

A hypergeometric sum is a finite or infinite sum of the form

$$\sum_k c_k,$$

where the ratio of successive terms  $\frac{c_{k+1}}{c_k}$  is a rational function of  $k$ . Such sums have been studied extensively for centuries and comprise a class of special functions with tremendous importance in mathematics. They frequently emerge in the study of number-theoretic problems of combinatorial type. They are notoriously hard to compute by traditional methods and an enormous database of related identities has been built up over hundreds of years. The work of Sister Mary Celine Fasenmyer 65 years ago introduced the first computerizable method to treat certain hypergeometric sums. This gave rise to ground-breaking future developments, namely Gosper's algorithm, Zeilberger's method of creative telescoping, the Wilf-Zeilberger method and Petkővsek's algorithm. We will study these important developments and attempt to use these methods in order to solve specific problems in combinatorial number theory.

Students participating in the project need to be mathematically mature and fluent in programming with Mathematica or Maple. A strong interest in combinatorial discrete mathematics is essential. Knowledge of elementary number theory and basic combinatorics is desired.