Homework #2, Due Friday, Sept. 30

1. #2.9 in Book
2. #3.10 in Book
3. Suppose \( \phi(\alpha) = (\alpha - 1)^2 \) for our step-length problem.
   (a) Determine the range of values for \( \alpha \) that satisfy the two Wolfe Conditions. Under what condition(s) on \( c_1 \) and \( c_2 \) do values of \( \alpha \) exist that satisfy both Wolfe Conditions? Under what condition(s) on \( c_1 \) and \( c_2 \) is the global minimizer \( \alpha = 1 \) included as a possible value?
   (b) Determine the range of values for \( \alpha \) that satisfies the Goldstein Condition:
   \[
   \phi(0) + (1 - c)\alpha\phi'(0) \leq \phi(\alpha) \leq \phi(0) + c\alpha\phi'(0),
   \]
   with \( 0 < c < \frac{1}{2} \)?
4. Let \( n \) be a positive integer and set \( f(x) = \sum_{i=1}^{n} f_i(x)^2 \) where
   \[
   f_i(x) = n - \sum_{j=1}^{n} (\cos x_j + i(1 - \cos x_i) - \sin x_i).
   \]
   Compute \( \nabla f \), \( \nabla^2 f \) and for \( x_0 = (\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n}) \), compute \( \nabla f(x_0) \) and \( \nabla^2 f(x_0) \).
5. Let
   \[
   A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}.
   \]
   Define \( f(x) = \frac{1}{2}x^T Ax \) for \( x \in \mathbb{R}^2 \), with unique minimizer \( x^* = (0, 0)^T \).
   (a) Determine all starting values \( x_0 \) with \( \|x_0\|_2 = 1 \) such that one step of Steepest Descent Method with Exact Line Search produces the exact answer \( x_1 = x^* \) (in exact arithmetic).
   (b) Determine all starting values \( x_0 \) with \( \|x_0\|_2 = 1 \) such that one step of Newton’s Method with Step-Length 1 produces the exact answer \( x_1 = x^* \) (in exact arithmetic).
   (c) (Bonus) Determine all starting values \( x_0 \) with \( \|x_0\|_2 = 1 \) such that \( x_k = x^* \) for some \( k \), \( 1 < k < 10 \), using the Steepest Descent Method with Exact Line Search.