Homework #1, Due Friday, Sept. 9

1. #2.1 in Book
2. #2.3 in Book
3. #2.8 in Book
4. #2.14 in Book
5. Let
$$A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \text{ and } b = \begin{bmatrix} 3 \\ -3 \end{bmatrix}.$$
Define $\phi(x) = \frac{1}{2}x^T Ax - b^T x$ for $x \in \Re^2$.
   (a) Find the minimizer of ϕ, i.e. $\hat{x}$ where $\phi(\hat{x}) \leq \phi(x)$.
   (b) Sketch the region in $\Re^2$ of all x such that $\phi(x) = 0$. Hint: think about the shape of the graph of ϕ. The result from part (c) might help also.
   (c) Find a change of variable of the form $x = Tw + c$ where T is a matrix and c a vector such that $\phi(x) = w_1^2 + w_2^2 + \alpha$, for some constant α. Hint: find the eigenvalues and eigenvectors of A.
6. Let $W(x) = (x-1)^2(x-2)^2 \cdots (x-5)^2$, which when multiplied out is
$$\begin{aligned} W(x) &= x^{10} - 30x^9 + 395x^8 - 3000x^7 + 14523x^6 - 46710x^5 \\ &+ 100805x^4 - 143700x^3 + 129076x^2 - 65760x + 14400 \end{aligned}$$
In the multiplied out form, estimate the value of $W(3)$ when evaluated on a computer that only stores 4 significant digits. (You can write a program to compute it, or you can come up with an answer justified by what we understand about roundoff error).