

Homework #5, Due Tuesday, March 14

Exercise 7: Quadratic Basis Functions in 2D

For the reference triangle T with vertices $z_1 = (0, 0)$, $z_2 = (1, 0)$ and $z_3 = (0, 1)$, construct the 6 functions on T which are used to form the quadratic basis functions. Let $z_{ij} = (z_i + z_j)/2$ and denote the functions you compute as ψ_i for those which satisfy $\psi_i(z_j) = \delta_{ij}$ and $\psi_i(z_{jk}) = 0$, and denote the functions as ψ_{ij} for those which satisfy $\psi_{ij}(z_{kl}) = \delta_{ik}\delta_{jl}$ and $\psi_{ij}(z_k) = 0$. (Note: $\delta_{ij} = 1$ if $i = j$ and $= 0$ otherwise).

Compute all 36 (actually just 21 distinct) values of $\int_T \nabla \psi_\alpha \cdot \nabla \psi_\beta dx$ as α and β range over all 6 possibilities. Put the results in a 6×6 grid.