

## MATH 572 Projectwork 1 - Test Problems

The examples below claim to satisfy the BVP

$$a(x)u''(x) + b(x)u'(x) + c(x)u(x) = f(x), \quad 0 < x < L$$

with  $u(0) = \alpha$  and  $u(L) = \beta$ .

1.  $a(x) = x$ ,  $b(x) = -x$ ,  $c(x) = x$ ,  $f(x) = x \sin(x)$ ,  $L = \frac{\pi}{2}$ ,  $\alpha = 1$ ,  $\beta = 0$ ,  
 $u(x) = \cos(x)$ .

(Note that this is the same as a constant coefficient problem if we divide through by  $x$ , thus this would show us what impact, if any, of the variable coefficients)

2.  $a(x) = x$ ,  $b(x) = -x$ ,  $c(x) = x$ ,  $f(x) = x^3 + 2x(1 - x)$ ,  $L = 4$ ,  $\alpha = 0$ ,  $\beta = 16$ ,  
 $u(x) = x^2$ .
3.  $a(x) = x^3$ ,  $b(x) = -x^2$ ,  $c(x) = -x$ ,  $f(x) = 2x^4 - 3x$ ,  $L = 2$ ,  $\alpha = 3$ ,  $\beta = 11$ ,  
 $u(x) = x^3 + 3$ .
4.  $a(x) = x^3$ ,  $b(x) = x^2$ ,  $c(x) = x$ ,  $f(x) = x^3$ ,  $L = 1$ ,  $\alpha = 0$ ,  $\beta = \frac{1}{5}$ ,  $u(x) = \frac{1}{5}x^2$ .
5.  $a(x) = x$ ,  $b(x) = -e^x$ ,  $c(x) = x$ ,  $f(x) = e^x \sin(x)$ ,  $L = \frac{\pi}{2}$ ,  $\alpha = 1$ ,  $\beta = 0$ ,  
 $u(x) = \cos(x)$ .
6.  $a(x) = (x^3 + 1)^3$ ,  $b(x) = 2x^2(x^3 + 1)^2$ ,  $c(x) = 12x(x^3 + 1)$ ,  $f(x) = 6(x^3 + 1)$ ,  
 $L = 1$ ,  $\alpha = 0$ ,  $\beta = \frac{3}{2}$ ,  $u(x) = \frac{3x^2}{x^3+1}$ .
7.  $a(x) = -x^3$ ,  $b(x) = x^2$ ,  $c(x) = 3x$ ,  $f(x) = 8x^2 + 3x$ ,  $L = 1$ ,  $\alpha = 1$ ,  $\beta = 4$ ,  
 $u(x) = x^3 + 2x + 1$ .

If you need to test with Neumann BCs, you can use these DEs and just compute the proper BCs.