Math 572 – Homework #10 – Due Thursday April 17

37. Determine the leading term of the error for these 2nd order methods:

$$\begin{aligned} y_{n+1} &= y_n + hf(t_n + \frac{1}{2}h, y_n + \frac{1}{2}hf(t_n, y_n)) \\ y_{n+1} &= y_n + \frac{h}{2}(f(t_n, y_n) + f(t_{n+1}, y_{n+1})) \\ y_{n+1} &= 4y_n - y_{n-1} + 2hf(t_{n+1}, y_{n+1}) \end{aligned}$$

38. For each of the following methods determine if they are convergent or not. If they are convergent, determine the order of the method.

    (a) $y_{n+1} = y_n + hf_{n+1}$

    (b) $y_{n+2} = \frac{3}{2}y_{n+1} - \frac{1}{2}y_n + hf_{n+1}$

    (c) $y_{n+2} = 3y_{n+1} - 2y_n + h(f_{n+2} - 2f_{n+1})$

39. (Derivation of Adams-Moulton)

    Determine the coefficients $\beta_0,\ \beta_1,\ \beta_2$ for the third order, 2-step Adams-Moulton method. Do this in two different ways:

    (a) Using the expression for the local truncation error and the conditions on the coefficients to force $C_0 = C_1 = C_2 = C_3 = 0$.

    (b) Using the relation

    $$y(t_{n+2}) = y(t_{n+1}) + \int_{t_{n+1}}^{t_{n+2}} f(s, y(s))\, ds.$$

    Interpolate a quadratic polynomial $p(t)$ through the three values $f(t_n, y^n),\ f(t_{n+1}, y^{n+1})$ and $f(t_{n+2}, y^{n+2})$ and then integrate this polynomial exactly to obtain the formula.

40. Determine and plot the region of absolute stability for the 2-step Adams-Moulton method. Is it A-stable, conditionally stable, or nowhere stable?