1. For each of the following 2nd order PDEs, determine domain(s) in the \((x, y)\)-plane where the PDE
(a) is hyperbolic, (b) is parabolic and (c) is elliptic.
\[
2u_{xx} + 4u_{xy} + 3u_{yy} - u = 0
\]
\[
yu_{xx} - 2u_{xy} + e^x u_{yy} + x^2 u_x - u = 0
\]

2. Let \(f(h) = a + O(h^p)\) and \(g(h) = b + O(h^q)\) with \(a \neq 0\), \(b \neq 0\) and \(0 \leq p \leq q\). Evaluate each of
the following expressions in the form of \(C + O(h^r)\) for some constants \(C\) and \(r\) and prove that your
expression is correct.
\[
(a) \ f(h) + g(h)
\]
\[
(b) \ f(h)g(h)
\]
\[
(c) \ f(h)/g(h)
\]

3. Let \(f\) be a smooth function of two variables \((t, x)\) and for given \(t_0\), \(x_0\) and \(h > 0\), let
\[
x_1 = x_0 + hf(t_0, x_0)
\]
\[
x_2 = x_0 + \frac{h}{2} (f(t_0 + h, x_1) + f(t_0 + h, x_2)).
\]
Use Taylor expansions to express \(x_2\) as a polynomial in \(h\) with coefficients depending on \(t_0\), \(x_0\) and
derivatives of \(f\) evaluated at \((t_0, x_0)\). Expand to at least the \(h^2\) term.

Hint: When there is an implicit expression it often works best to assume you have an expansion, but
with unknown coefficients. Then you can use the given expressions to solve for the coefficients. In
this case, you might try \(x_2 = x_0 + Ah + Bh^2 + O(h^3)\).

4. In this problem the norms are all grid function norms (see A.5). For each of the following inequality
representing the norm equivalence of various grid function norms, construct a vector \(e = (e_1, \ldots, e_N)\)
which makes the inequality an equality. Note that \(h = \frac{b-a}{N}\).
\[
h\|e\|_\infty \leq \|e\|_1 \leq Nh\|e\|_\infty
\]
\[
\sqrt{h}\|e\|_\infty \leq \|e\|_2 \leq \sqrt{Nh}\|e\|_\infty
\]
\[
\sqrt{h}\|e\|_2 \leq \|e\|_1 \leq \sqrt{Nh}\|e\|_2.
\]
(There are 6 equalities you need to find \(e\)s for. You can use a different \(e\) for each one, but each should
depend on \(N\)).

If you don’t have the text yet, here are the norms:
\[
\|e\|_1 = h \sum_i |e_i|
\]
\[
\|e\|_2 = \sqrt{h} \sqrt{\sum_i |e_i|^2}
\]
\[
\|e\|_\infty = \max_i |e_i|.
\]

5. (Extra) Prove the following inequality showing the norm equivalence of the 1 and \(\infty\) grid function
norms:
\[
h\|e\|_\infty \leq \|e\|_1 \leq Nh\|e\|_\infty.
\]