Math 571 – Fall 2007 – Homework #9
Due Thursday, November 8

You can use any written resources, but please work alone.

28. The Hessenberg QR-Step:

    Given H in upper Hessenberg form, we overwrite H with RQ where $H = QR$ is the QR factorization of H.

    For $k = 1$ to $m - 1$
    Find θ_k such that
    $$\begin{pmatrix} c_k & s_k \\ -s_k & c_k \end{pmatrix} \begin{pmatrix} h_{kk} \\ h_{k+1,k} \end{pmatrix} = \begin{pmatrix} \alpha_k \\ 0 \end{pmatrix}$$
    where $c_k = \cos(\theta_k)$ and $s_k = \sin(\theta_k)$ and α_k is any number
    For $j = k$ to m
    $$\begin{pmatrix} h_{kj} \\ h_{k+1,j} \end{pmatrix} = \begin{pmatrix} c_k & s_k \\ -s_k & c_k \end{pmatrix} \begin{pmatrix} h_{kj} \\ h_{k+1,j} \end{pmatrix}$$
    End For
    End For
    For $k = 1$ to $m - 1$
    For $i = 1, \ldots, k + 1$
    $$\begin{pmatrix} h_{ik} & h_{i,k+1} \end{pmatrix} = \begin{pmatrix} h_{ik} & h_{i,k+1} \end{pmatrix} \begin{pmatrix} c_k & -s_k \\ s_k & c_k \end{pmatrix}$$
    End For
    End For

    Rewrite this algorithm in the case that H is symmetric and upper Hessenberg (and thus tridiagonal). Write out the matrix multiplications and take advantage of the knowledge that H is tridiagonal before and after this process. (See #16 from HW#4 as to how to compute c_k and s_k (Note the sign change)). Show that the resulting algorithm performs this step in some multiple of m flops.

29. (Continuation of the Previous Problem)

    Since H is tridiagonal and symmetric, we can store it as just two m vectors a and b where a contains the diagonal elements and b the subdiagonal elements.

    (a) Rewrite the previous algorithm using this new structure for H. Try to minimize any additional storage you use during the process.

    (b) Now to get the full version of QR with Shifts. Add the calculation of the Wilkinson Shift and the use of that shift to the algorithm.