1. For the copier: Let there be three states for each hour: (1) idle, (2) in use, and (3) broken. We’ll assume that a typical use takes less than one hour, so that without any other factors, 40% of the time it is idle and 60% of the time it is in use. We’ll also assume that being broken and then repaired doesn’t change the demand and that after it is repaired it becomes idle. (I think it would be better to say it has a 40% chance of being idle and a 60% chance of being in use after being repaired).

Since we expect to change some of the probabilities involved, we’ll introduce them as variables. Let $u$ be the probability of being in use (after being idle), $i$ the probability of being idle (after being in use), $b$ be the probability of being broken (after being in use) and $r$ the probability of being repaired (after being broken). Then we have the corresponding transition matrix:

$$T = \begin{pmatrix} 1 - u & i & r \\ u & 1 - i - b & 0 \\ 0 & b & 1 - r \end{pmatrix}.$$  

Since we will be interested in the stable distribution (fixed point) for the Markov process driven by this $T$, we can solve symbolically for $\hat{\pi}$ such that $T\hat{\pi} = \hat{\pi}$. We also know that the components of $\hat{\pi}$ sum to 1, so with a bit of algebra we get

$$\hat{\pi} = \begin{pmatrix} \frac{r(i+b)}{r(i+b)+u(r+b)} \\ \frac{ru}{r(i+b)+u(r+b)} \\ \frac{bu}{r(i+b)+u(r+b)} \end{pmatrix}.$$  

The quantity of interest is the probability that the copier is not broken, i.e. it is either idle or in use, which in our stable distribution works out to be

$$p = \frac{r(i+b) + ru}{r(i+b)+u(r+b)} = \frac{r(i+b+u)}{r(i+b)+u(r+b)}.$$  

Now we can easily address the various scenarios:

(a) Original Case: The information given does not uniquely define the constants as it is not clear whether or not the 40/60 split between idle/in use includes the repair time or not. Some parts are fairly clear. If the average time between becoming broken out of being used is 40 hours, then $b = 1/40$. Also if the average repair time is 4 hours, then $r = 1/4$. The key is to realize these transition probabilities are about the transition from one specific state to another and not about how much time is spent out of the total time in these states. To assign quantities to $i$ and $u$ I assumed that the 40/60 ignored the repair time so that $i = 0.4$ and $u = 0.6$, so that without the broken state, we’d have the perfect 40/60 split.

With these values $p = 0.9447...$ meaning that in a 60 hour week the copier is unavailable 3h19m.

(b) Upgrade Service: same as (a) but now the average repair time is 2 hours, so $r = 1/2$. Then $p = 0.9716...$. This means in a 60 hour week, it is broken only 1h42m.
(c) Upgrade Copier: same as (a) but now the average time before being broken is 70 hours, thus $b = 1/70$. Then $p = 0.9673 \ldots$. This means in a 60 hour week, it is broken only 1h58m.

(d) Go Paperless: same as (a) but now the usage is decreased to 60%, thus $u = 0.4$ and $i = 0.6$. Then $p = 0.9624 \ldots$. This means in a 60 hour week, it is broken only 2h15m. However since we are only using the copier 40% of the time versus 60% in the other cases, this is about the same as it being broken 1h30m.

The best choice from pure availability is to upgrade the service contract as it saves 1h37m in down time per week. You could argue that since there is less demand when you go paperless (d) then although the down time is larger, there is really less of an impact with option (d).

To evaluate whether it is worth making any of these changes, we’d have to look at the cost to the company of the down time. For example if it costs the company $100 per hour of lost copier time, then the dollar cost per year of each of the 4 scenarios (Original, Service, Copier, Paperless) are $17,253, $8,892, $10,202 and $11,731, respectively. You could argue that the last scenario should have lower loss cost ($67) which would make its yearly cost $7,859. Then we could first look at the cost of each solution versus the savings which is best.

Suppose it will cost $1000 to buy a scanner and then $2000 per year to run and maintain it. Then over a 5-year period we’d spend $11,000 but save $27,610 for a net profit of $16,610. If instead we bought a new copier at a cost of $15,000, then we’d have spent $15,000 but save $35,255 over 5 years for a profit of $20,225. The second option has a higher profit but it will take over 2 years to pay back the original investment. The second option has a lower overall profit but the initial cost would be paid back very quickly and the department would see a portion of the profit in the first year.