Matrix Manipulation Examples for Markov Processes

Example 1: 4 Room Party

The transition matrix (constructed in class) was

$$T = \begin{pmatrix} 0.8 & 0 & 0.3 & 0 \\ 0 & 0.5 & 0.4 & 0 \\ 0.2 & 0.5 & 0.1 & 0.1 \\ 0 & 0 & 0.2 & 0.9 \end{pmatrix}.$$

The eigenvalues (with their eigenvectors) are:

$$1.00, \begin{pmatrix} 0.2830 \\ 0.1509 \\ 0.1887 \\ 0.3744 \end{pmatrix} \qquad 0.87, \begin{pmatrix} 0.5503 \\ 0.1371 \\ 0.1264 \\ -0.8139 \end{pmatrix} \qquad 0.67, \begin{pmatrix} 0.6711 \\ -0.6399 \\ -0.2798 \\ 0.2486 \end{pmatrix} \qquad -0.24, \begin{pmatrix} 0.2427 \\ 0.4541 \\ -0.8444 \\ 0.1477 \end{pmatrix}$$

Example 2: SIR Disease Model

The transistion matrix is

$$T = \begin{pmatrix} 0.6 & 0 & 0 \\ 0.3 & 0.9 & 0 \\ 0.1 & 0.1 & 1 \end{pmatrix}.$$

The eigenvalues (with their eigenvectors) are:

$$1.00, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad 0.90, \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \qquad 0.60, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

T is block form:

$$T = \begin{pmatrix} A & 0 \\ B & I_1 \end{pmatrix},$$

where

$$A = \begin{pmatrix} 0.6 & 0 \\ 0.3 & 0.9 \end{pmatrix}, B = \begin{pmatrix} 0.1 & 0.1 \end{pmatrix},$$

0 is a matrix of all zeros and I_1 is the 1×1 identity. Then

$$F = (I_2 - A)^{-1} = \begin{pmatrix} 2.5 & 0 \\ 7.5 & 10.0 \end{pmatrix} \text{ and } BF = \begin{pmatrix} 1.0 & 1.0 \end{pmatrix}.$$