Math 411 – Group Project

1. The model(s) you use must be discrete and stochastic. In particular it should involve a Markov Process and have at least 5 possible states.

2. Since forming the transition matrix is the most critical part of the model, carefully justify your choices for the values you use.

3. In analyzing the model(s), you can use computation to produce values, but you should also use analytic techniques to either find the steady state distribution (if there is one) or probabilities and transition times to absorbing state(s) (if there are any).

4. If it makes sense, introduce some values for the states so that you can compute the expected value.

Ideas:

- Board Game. Take or make-up any game which uses a spinner or dice or any random type event to make a move and model it or some aspect of it. For example: Chutes and Ladders, Cootie or one battle in Risk. You should answer questions like:

  1. What is the probability of winning after N turns?
  2. What is the average position after N turns?
  3. What is the average number of turns needed to win?
  4. If the player has some options, what is his or her best strategy?
  5. If the rules are changed in some way, how does that change the results?

- World Series (or any Best of 7 Series). Study the probability of each team winning a best of 7 series, i.e. the first to win 4 games, wins. You can include home field advantage or any other relevant variation in the probabilities. Some questions you might answer:

  1. For various choices of probabilities, what is the expected number of games that will be played?
  2. What are the probabilities for comebacks? I.e. what is the probability of being down 0-2 and winning 4-3 or 4-2?

- Multistage Disease. Consider a disease that has multiple stages. For example Chicken Pox has at least 3 stages: infected with no signs, infected, serious infection. No matter what the disease you will also have the stages: susceptible, immune and dead.

  Remember: If it takes M days to go through a stage, then the probability each day of exiting the stage is $1/M$.

  Here are some questions to consider:

  1. When are the most people infected?

2. What is the average length of time one has the disease?
3. What is the overall death rate?
4. What is the effect of vaccination? That is, if more and more people start out immune, how does that effect the infection rate and the death rate?

- Groups in a Population. One can divide a population into different groups based on various criterion, eg. education level or political preferences. From one generation to the next, there will be some transition between these classes. Model this transition in a case with 5 or more classes. You might be able to find values for the transition rates, or you can just make them up. Some questions you might answers:

  1. Is there a stable distribution between the classes?
  2. Score each class according to their need for some service that society provides (like education) and determine if the need increases or decreases with each generation.
  3. Score each class according to their contribution to society (like taxes paid) and determine if the overall contribution increases or decreases.
  4. Suppose the transitions depend upon the availability of some service (like education). Study the need and contribution under different availabilities of this resource.

- Diffusion. As in class define rules for the random movement between compartments. Using a large number of compartments, consider the following questions:

  1. What is the distribution after N turns?
  2. Is there a steady state distribution?
  3. If we change the layout (or the rules) how does the answers to 1. and 2. change?
  4. What is the average length of time for a particle to travel from one compartment to another?

Deadlines:
Project Proposal: Monday, October 12
Presentation: October 28, 30
Report: Friday, October 30
Project Proposal: A 1-2 page report containing

1. List of the members of your group
2. The area or subject of your model
3. A description of the context for your questions, including any background information or data that you feel might be relevant (you don't have to have the data yet, just write out what you think you might need)
4. 3-5 questions that you are considering trying to answer; be specific
5. (Optional) A list of questions for me about your model or the process

I'd suggest you give some time for the members to think individually about the project and then have a meeting (or email exchange) discussing the ideas. Appoint someone to do the write-up and have them circulate it among the members before it is turned in. It is okay if you change your mind later about what your project is about.
Presentation:

1. Electronic format (Powerpoint or a PDF); email to me before class or bring it on a USB drive.
2. You'll also need to hand-in a hard copy of your presentation.
3. Time limits: 10 minutes maximum, 8 minutes minimum
4. Should contain a question, a model, some analysis and a result. Does not (and should not) contain all the work you've done.
5. Will be graded 50% on the content related to a consistent and comprehensive flow from question to answer and 50% on presentation quality (timing, clarity and quality of graphics/format)

Report:

1. Typed. No page limit, but it should be complete. Write as for a well-educated audience, but who does not necessarily have a depth of background in either the context or the mathematics.
2. Will be graded 60% on format and 40% on content. The required elements are:
   (a) Abstract or Executive Summary. A less than one page summary of your project giving at least some background, at least one question and at least one result.
   (b) Question and Background. Develop the context for the project. State the question(s) you will be addressing. State the assumptions and elements that are important in the development of the model.
   (c) Model Building. Develop the model based on the assumptions and gathered data.
   (d) Analysis. Solve the model (by whatever means). Analyze the answers and, if possible, check their validity and sensitivity to the data.
   (e) Results. Interpret the solutions to the model(s) and relate back to the original question. Also reflect on the quality of the overall process and discuss the strengths and weaknesses of the approach.
   (f) Bibliography. Reference any sources you used, including class notes, the class text, internet sites, etc.