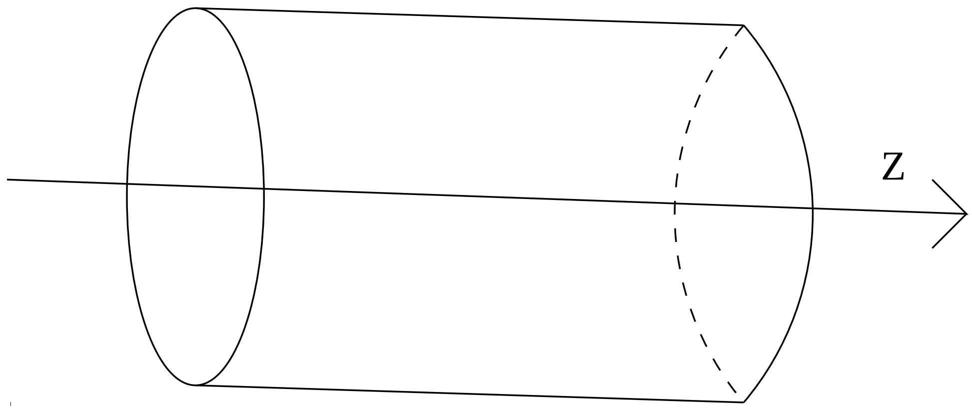
Math 411 – Flow and Diffussion in a Tube

This is a summary (with notes) of the FDIT (Flow and Diffusion in a Tube) model. The ideal setting is the flow of a medium in a tube, where the medium contains a suspension, eg. sand in water in a pipe. However, this model is applicable to a wide range of situations.

Here is a tube:



We use z to denote distance along the tube. By assumption all values depend only on z and t (time). The model we use for the flow is:

$$\frac{\partial(A\phi)}{\partial t} + AK\phi + \frac{\partial H}{\partial z} = AC + q.$$

All variables in this expression unless otherwise stated can be dependent on z and t, and possibly ϕ. Also, all variables are considered to be non-negative. The variables are:

1. A is the cross-sectional area of the tube, assumed constant but could vary slightly as a function of z. The cross-section does not have to be a circle, just essentially uniform.

2. ϕ is the concentration of the suspension.

3. K is the relative reaction rate for the internal destruction of suspension.

4. H is the flux (flow of suspension) in the z direction, its exact form depends on the type of quantity ϕ is.

   Let w be the velocity of the medium (in the z direction). The forms of H are:

   (a) When ϕ measures something tied to the medium (like density).
   $H = Aw\phi$.

   (b) When ϕ measures something which is independent of the medium (like sand in water).
   $H = A(w\phi - \gamma\frac{\partial\phi}{\partial z})$, where γ is the coefficient of diffussion.

   (c) When ϕ measures the momentum of the medium. ($\phi = \rho w$, ρ is the density)
   $H = A(\rho w^2 + p - \frac{\partial(\mu w)}{\partial z})$, where p is the pressure and μ is the viscosity of the medium.

5. C is the reaction rate for the internal creation of suspension. Units: amt/time.

6. q is the injection rate through the sides of the tube. $q < 0$ means suspension is removed.