Math 241 – Sample Exam 1

Disclaimer: This sample exam may or may not be like the actual exam. The actual exam may contain material not on this sample exam. Relying too much on how well you can do on this sample exam may be detrimental to your performance on the actual exam.

1. Let $\vec{\mathbf{a}} = \langle 1, -2, 1 \rangle$ and $\vec{\mathbf{b}} = \langle 2, 2, -1 \rangle$. Compute:
   (a) $|\vec{\mathbf{a}} - \vec{\mathbf{b}}|$
   (b) the scalar projection $\text{comb}_{\vec{\mathbf{a}}}(\vec{\mathbf{b}})$
   (c) the vector projection $\text{proj}_{\vec{\mathbf{a}}}(\vec{\mathbf{b}})$
2. Two people pull a sled from the point $A(-1, -2, 0)$ to the point $B(1, 3, 0)$. One pulled with constant force $\vec{\mathbf{F}}_1 = 2\vec{\mathbf{i}} + 7\vec{\mathbf{j}} + \vec{\mathbf{k}}$ while the other pulled with constant force $\vec{\mathbf{F}}_2 = 9\vec{\mathbf{j}} + 3\vec{\mathbf{k}}$. How much work did each do?
3. Find a unit vector which is orthogonal to both $\vec{\mathbf{a}} = \vec{\mathbf{i}} + \vec{\mathbf{k}}$ and $\vec{\mathbf{b}} = \vec{\mathbf{j}} + \vec{\mathbf{k}}$.
4. Give in parametric form the equation for the line through the points $P(2, 3, -4)$ and $Q(-1, 1, 1)$. Find the point where the line intersects the plane $x + 2y - z = 4$.
5. Consider the two planes $3x - 2y + z = 5$ and $9x - 6y + 3z = -2$.
   (a) Show that the planes are parallel.
   (b) Give the symmetric form of the equation of the line which is orthogonal to both planes and goes through the origin.
   (c) Find the distance between the planes.
6. Describe the domain of the function
$$f(x, y) = \frac{\sin(xy)}{x^2 - xy}.$$
7. Sketch or describe the graph of $z = f(x, y) = \cos(y)$.
8. Sketch or describe the set S given in spherical coordinates as
$$S = \{(\rho, \theta, \phi) : \rho = 2, \frac{\pi}{4} \leq \phi \leq \frac{3\pi}{4}\}$$

Solutions:

1. (a) $|\vec{\mathbf{a}} - \vec{\mathbf{b}}| = |\langle -1, -4, 2\rangle| = \sqrt{1 + 16 + 4} = \sqrt{21}$.

   (b) $\frac{\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}}{|\vec{\mathbf{a}}|} = \frac{-3}{\sqrt{6}}$

   (c) $\frac{\vec{\mathbf{a}}\cdot\vec{\mathbf{b}}}{|\vec{\mathbf{a}}|^2}\vec{\mathbf{a}} = \frac{-3}{\sqrt{6}}\frac{\vec{\mathbf{a}}}{\sqrt{6}} = \langle -\frac{1}{2}, 1, -\frac{1}{2}\rangle$.

2. The sled's movement is given by $\vec{\mathbf{a}} = \vec{AB} = \langle 2, 5, 0\rangle$, (in meters) so the first person works $\vec{\mathbf{a}} \cdot \vec{\mathbf{F}}_1 = 39J$ and the second works $\vec{\mathbf{a}} \cdot \vec{\mathbf{F}}_2 = 45J$.

3. Let $\vec{\mathbf{c}} = \vec{\mathbf{a}} \times \vec{\mathbf{b}} = -\vec{\mathbf{i}} - \vec{\mathbf{j}} + \vec{\mathbf{k}}$. Then the unit vector is $\frac{\vec{\mathbf{c}}}{|\vec{\mathbf{c}}|} = -\frac{1}{\sqrt{3}}\vec{\mathbf{i}} - \frac{1}{\sqrt{3}}\vec{\mathbf{j}} + \frac{1}{\sqrt{3}}\vec{\mathbf{k}}$, or the negative of this vector.

4. $\vec{\mathbf{v}} = \vec{PQ} = \langle -3, -2, 5\rangle$ so the parametric equations are: $x = 2 - 3t$, $y = 3 - 2t$ and $z = -4 + 5t$. For intersection we have

   $$(2 - 3t) + 2(3 - 2t) - (-4 + 5t) = 4 \text{ or } t = \frac{2}{3}$$

   For the point we get $(0, \frac{5}{3}, -\frac{2}{3})$.

5. (a) Their normals are $\vec{\mathbf{n}}_1 = \langle 3, -2, 1\rangle$ and $\vec{\mathbf{n}}_2 = \langle 9, -6, 3\rangle$. Clearly $\vec{\mathbf{n}}_2 = 3\vec{\mathbf{n}}_1$, also $\vec{\mathbf{n}}_1 \times \vec{\mathbf{n}}_2 = \vec{0}$, so the planes are parallel.

   (b)

   $$\frac{x}{3} = \frac{y}{-2} = \frac{z}{1}$$

   (c) $P(0, 0, 5)$ is on the first plane and the distance from P to the second plane is

   $$\frac{|9(0) - 6(0) + 3(5) - (-2)|}{\sqrt{81 + 36 + 9}} = \frac{17}{\sqrt{126}}$$

6. Domain: $D = \{(x, y) \mid x^2 - xy \neq 0\}$. Since $x^2 - xy = x(x - y)$, the domain D is all points in the x, y-plane, except those on the lines $x = 0$ and $x = y$.

7. The graph of $z = \cos(y)$, looks like a corregated tin roof. The traces for $x = k$ is the cosine curve (same for all k). So the 'valleys' of the roof run parallel to the x-axis.

8. S is a surface in $\mathbf{R}^3$ which is a 'belt' around a sphere. The sphere has radius 2 and is centered at the origin. The belt extends from 45° above the equator to 45° below the equator.