Math 142 – Quiz 3 – Solutions

I. Make the substitution, simplify but don't evaluate.

(a) With $\theta = \pi x$, $d\theta = \pi\, dx$, $\theta = \pi$ when $x = 1$ and $\theta = -\pi$ when $x = -1$. Thus we get

$$\int_{-1}^{1} \cos^2(\pi x)\, dx = \frac{1}{\pi}\int_{-\pi}^{\pi} \cos^2\theta\, d\theta.$$

(b) With $u = 2 - 3x^3$, $du = -9x^2\, dx$ or $-\frac{1}{9}du = x^2 dx$, $u = 26$ when $x = -2$ and $u = -22$ when $x = 2$. Thus we get

$$\int_{-2}^{2} 7x^2(2-3x^3)^4\, dx = -\frac{7}{9}\int_{26}^{-22} u^4\, du. = \frac{7}{9}\int_{-22}^{26} u^4\, du.$$

(c) With $u = e^x$, $du = e^x\, dx$. By noting that $e^{2x} = e^x \cdot e^x$, we get

$$\int e^{2x}\sqrt{1+e^{2x}}\, dx = \int u\sqrt{1+u^2}\, du.$$

(d) With $x = 3\sin\theta$, $dx = 3\cos\theta\, d\theta$ and $9 - x^2 = 9 - 9\sin^2\theta = 9\cos^2\theta$, so we get

$$\int \frac{x^2}{\sqrt{9-x^2}}\, dx = \int \frac{(3\sin\theta)^2(3\cos\theta)}{\sqrt{9\cos^2\theta}}\, d\theta = 9\int \sin^2\theta\, d\theta.$$

(e) With $u = 2 - x$, $du = -dx$, $x = 2 - u$, $u = 0$ when $x = 2$ and $u = 2$ when $x - 0$. We then get

$$\int_0^2 x\sqrt{2-x}\, dx = -\int_2^0 (2-u)\sqrt{u}\, du = \int_0^2 (2-u)\sqrt{u}\, du.$$

(f) With $u = \ln x$, $x = e^u$, $dx = e^u du$, so

$$\int \ln x\, dx = \int ue^u\, du.$$

II. Evaluate and label with rule number.

(a) Using #25 with $a = \sqrt{8}$, we get

$$\int \frac{1}{\sqrt{8+y^2}}\,dy = \ln(y + \sqrt{8+y^2}) + C.$$

(b) Using #97 then #96, with $a = -2$, we get

$$\int x^2 e^{-2x}\,dx = -\frac{1}{2}x^2 e^{-2x} + \int x e^{-2x}\,dx = (-\frac{1}{2}x^2 - \frac{1}{2}x - \frac{1}{4})e^{-2x} + C.$$

(c) Using #99 with $a = -1$, $b = 2$, we get

$$\int e^{-x}\cos(2x)\,dx = \frac{e^{-x}}{5}(-\cos(2x) + 2\sin(2x)) + C.$$

(d) First substituting $u = e^t$ with $du = e^t\,dt$ we get

$$\int e^t \sin^3(e^t)\,dt = \int \sin^3(u)\,du.$$

Then apply #67 (and substitute back in for u) to get

$$\int e^t \sin^3(e^t)\,dt = -\frac{1}{3}(2 + \sin^2 e^t)\cos e^t + C.$$

(e) First note $\cot x = \frac{\cos x}{\sin x}$, so we can substitute $u = \sin x$, $du = \cos x\,dx$ to get

$$\int \frac{\cot x}{\sqrt{1+2\sin x}}\,dx = \int \frac{1}{u\sqrt{1+2u}}\,du.$$

Then apply #57 with $a = 1$, $b = 2$ (and sub back in for u), to get

$$\int \frac{\cot x}{\sqrt{1+2\sin x}}\,dx = \ln\left|\frac{\sqrt{1+2\sin x} - 1}{\sqrt{1+2\sin x} + 1}\right| + C.$$

(f) First rewrite $s^2 + 2s - 1 = s^2 + 2s + 1 - 2 = (s+1)^2 - 2$, then take $u = s + 1$, $du = ds$ to get

$$\int \frac{1}{\sqrt{s^2+2s-1}}\,ds = \int \frac{1}{\sqrt{u^2-2}}\,du.$$

Then apply #43 with $a = \sqrt{2}$, and sub for u, to get

$$\int \frac{1}{\sqrt{s^2+2s-1}}\,ds = \ln\left|s + 1 + \sqrt{s^2+2s-1}\right| + C.$$