

Math 142 – Quiz 2 – Solutions

1. (a) $g'(x) = -(x^2 - 1)e^{-x^2}$

(b) $g'(x) > 0$ when $-(x^2 - 1) > 0$ or $x^2 < 1$, thus for $x \in [0, 1)$.

2.

(a) $\int \cos^2 3x \, dx = \frac{1}{2}x + \frac{1}{12} \sin 6x + C$

Trig identity $\cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$ and sub $u = 6x$

(b) $\int s^2 e^{2s} \, ds = \left(\frac{1}{2}s^2 - \frac{1}{2}s + \frac{1}{4}\right)e^{2s} + C$ Integration by parts twice

(c) $\int 3x(2 - 4x^3)^2 \, dx = 6x^2 - \frac{48}{5}x^5 + 6x^8 + C$ Multiply out

(d) $\int \frac{\sin \sqrt{x}}{\sqrt{x}} \, dx = -2 \cos \sqrt{x} + C$ Sub $u = \sqrt{x}$

(e) $\int t\sqrt{1-t} \, dt = -\frac{2}{3}(1-t)^{3/2} + \frac{2}{5}(1-t)^{5/2} + C$

Sub $u = 1 - t$ or integration by parts $u = t$, $dv = \sqrt{1-t} \, dt$

(f) $\int \ln x \, dx = x \ln x - x + C$ Parts $u = \ln x$, $dv = dx$

(g) $\int x \cos(\pi x) \, dx = \frac{x}{\pi} \sin \pi x + \frac{1}{\pi^2} \cos \pi x + C$ Parts $u = x$, $dv = \cos \pi x \, dx$

(h) $\int 4y^2(9 - 3y^3)^5 \, dy = -\frac{2}{27}(9 - 3y^3)^6 + C$ Sub $u = 9 - 3y^3$

(i) $\int \frac{x^2}{\sqrt{1-x^2}} \, dx = \frac{1}{2} \sin^{-1} x - \frac{1}{2}x\sqrt{1-x^2} + C$

Trig Sub $x = \sin \theta$

(j) $\int \cos^5 \theta \sin^2 \theta \, d\theta = \frac{1}{3} \sin^3 \theta - \frac{2}{5} \sin^5 \theta + \frac{1}{7} \sin^7 \theta + C$

Trig Identity $\cos^2 \theta = 1 - \sin^2 \theta$, then sub $u = \sin \theta$

(k) $\int x\sqrt{x^2-1} \, dx = \frac{1}{3}(x^2-1)^{3/2} + C$ Sub $u = x^2 - 1$