

Math 142 – Quiz 1 – Solutions

1. The area as a (a) a limit: $\lim_{n \rightarrow \infty} \sum_{i=1}^n e^{-x_i^2} \Delta x$, (where $\Delta x = 3/n$, $x_i = -1 + i\Delta x$, could use x_i^* instead)
(b) an integral: $\int_{-1}^2 e^{-x^2} dx$

2.

(a) $\int_1^5 f(x) dx = \int_1^2 f(x) dx + \int_2^5 f(x) dx = 5 + 4 = 9$

(b) $\int_1^2 f(x) dx = \int_1^3 f(x) dx - \int_2^3 f(x) dx = 7 - 2 = 5$

(c) $\int_1^3 (2 - 3f(x)) dx = \int_1^3 2 dx - 3 \int_1^3 f(x) dx = 2(3 - 1) - 3 \cdot 7 = -17$

3. Find the general indefinite integral for

$$\int (x^2 + 1)^2 dx = \int (x^4 + 2x^2 + 1) dx = \frac{1}{5}x^5 + \frac{2}{3}x^2 + x + C$$

4. Evaluate the following definite integrals:

(a) $\int_{-3}^2 2x^2 dx = \frac{2}{3}x^3 \Big|_{-3}^2 = \frac{16}{3} - (-18) = \frac{70}{3}$

(b) $\int_0^{\pi/2} \sin(\theta) d\theta = -\cos(\theta) \Big|_0^{\pi/2} = -\cos(\pi/2) - (-\cos(0)) = 1$

(c) $\int_1^e \left(\frac{3}{x} + 4e^x \right) dx = 3 \ln|x| + 4e^x \Big|_1^e = 3 \ln e + 4e^e - 3 \ln 1 - 4e^1 = 3 + 4e^e - 4e$

(d) $\int_4^9 \frac{1-x}{\sqrt{x}} dx = \int_4^9 x^{-1/2} - x^{1/2} dx = 2x^{1/2} - \frac{2}{3}x^{3/2} \Big|_4^9 = (6 - 8) - (4 - \frac{16}{3}) = -\frac{32}{3}$

5. $v(t) = 2t - 4$

$$\text{Position} = \int_0^3 2t - 4 dt = t^2 - 4t \Big|_0^3 = 9 - 12 = -3$$

$$\text{Distance} = \int_0^3 |2t - 4| dt = \int_0^2 4 - 2t dt + \int_2^3 2t - 4 dt = 4 + 1 = 5$$