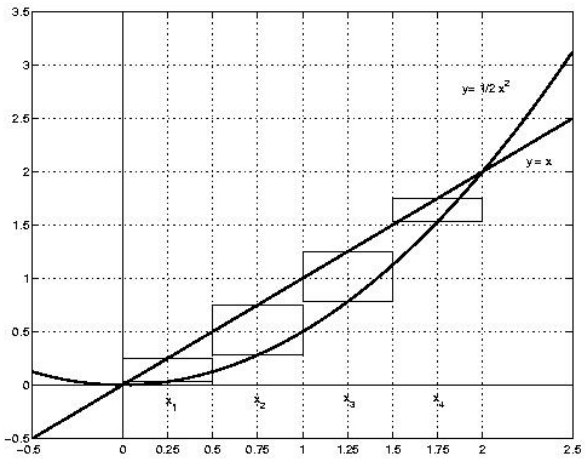
John B. Student Homework Project #2

Find the area of the region below the curve $f(x) = x$ and above the curve $g(x) = 1/2x^2$.



Approximate area:

Using 4 rectangles and midpoint values:

$$s_4 = \sum_{i=1}^{4} (f(x_i) - g(x_i))\Delta x.$$

With $x_1 = 1/4$, $x_2 = 3/4$, $x_3 = 5/4$, $x_4 = 7/4$ and $\Delta x = 1/2$, we have

$$\begin{aligned} s_4 &= (1/4 - 1/2(1/4)^2)(1/2) + (3/4 - 1/2(3/4)^2)(1/2) \\ &+ (5/4 - 1/2(5/4)^2)(1/2) + (7/4 - 1/2(7/4)^2)(1/2) \\ &= 11/16 \qquad (0.6875). \end{aligned}$$

Analysis:

$$\begin{aligned} \text{Area} &= \lim_{n\to\infty} s_n && \text{Definition of Area} \\ &= \int_0^2 (x - 1/2x^2)\,dx && \text{Def. of Integral} \\ &= F(2) - F(0) && \text{FTC} \end{aligned}$$

Evaluation (FTC):

An antiderivative of $h(x) = x - 1/2x^2$ is

$$F(x) = 1/2x^2 - 1/6x^3.$$

Check:

$$F'(x) = 1/2(2x) - 1/6(3x^2) = x - 1/2x^2.$$

So

$$\begin{aligned} \text{Area} &= F(2) - F(0) \\ &= (4/2 - 8/6) - (0 - 0) = 2/3. \end{aligned}$$

Evaluation (limit):

Using the attached MATLAB program:

k	s_n	-log(h)	-log(err)
1	7.500000e-01	-0.000	0.903
2	6.875000e-01	0.301	1.505
3	6.718750e-01	0.602	2.107
4	6.679688e-01	0.903	2.709
5	6.669922e-01	1.204	3.311
6	6.667480e-01	1.505	3.913
7	6.666870e-01	1.806	4.515
8	6.666718e-01	2.107	5.118
9	6.666679e-01	2.408	5.720
10	6.666670e-01	2.709	6.322