Math 300 – Order Axiom and Absolute Value Problems

Definition: (1) $a > b$ means $b < a$. (2) $a \leq b$ means $a < b$ or $a = b$. (3) $a < b < c$ means $a < b$ and $b < c$.

C-1. Find all $x \in \mathbb{R}$ for which the inequality $(x+1)(x-2) > 4$ is true.

C-2. Find all $x \in \mathbb{R}$ for which the inequality $\frac{x}{x+2} < 3$ is true.

C-3. Find all $x \in \mathbb{R}$ for which the inequality $\frac{x-1}{2x-1} < -1$ is true.

C-4. Find all $x \in \mathbb{R}$ for which the inequality $\frac{x-2}{x^2-4} > \frac{1}{2}$ is true.

C-5. Find all $x \in \mathbb{R}$ for which the inequality $1 < \frac{x-2}{(x-3)(x-1)} < 2$ is true.

In the following assume all are elements of an ordered field and if specific numbers are involved then the field is $\mathbb{R}$.

P-1. $a > 0$ if and only if $-a < 0$.

P-2. $-1 < 0 < 1$.

P-3. If $a \neq 0$ then $a^2 > 0$.

P-4. If $a < b$ and $c < 0$, then $a \cdot c > b \cdot c$.

P-5. If $a < b$ and $c < d$, then $a + c < b + d$.

P-6. If $0 < a < b$ and $0 < c < d$, then $a \cdot c < b \cdot d$.

P-7. If $a < b$, $b \geq 0$, and $0 \leq c \leq d$, then $a \cdot c \leq b \cdot d$.

P-8. If $a > 0$, then $a^{-1} > 0$.

P-9. If $a \cdot b > 0$ then either $a > 0$ and $b > 0$, or, $a < 0$ and $b < 0$.

P-10. If $a > 0$, then $\exists\ \epsilon > 0$ such that $a > \epsilon$.

P-11. Let $a, b \in \mathbb{R}$. $a < b + \epsilon$ holds for all $\epsilon > 0$ if and only if $a \leq b$.

P-12. $a > b - \epsilon$ holds for all $\epsilon > 0$ if and only if $a \geq b$.

Definition: For $a \in \mathbb{F}$ where $\mathbb{F}$ is an ordered field, the *absolute value of* a (written $|a|$) evaluates as follows:

$$|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}$$

C-1. Solve for all x which satisfy $|x+2| + |x-1| = 4$.

C-2. Solve for all x which satisfy $||x-1|-1| < 2$.

C-3. Sketch a graph of the set of all (x, y) which satisfy $|y| = x^2$.

C-4. Sketch a graph of the set of all (x, y) which satisfy $|x| + |y| = 1$.

C-5. Sketch a graph of the set of all (x, y) which satisfy $|x| - |y| \leq 1$.

Assume all are elements of an ordered field.

P-1. $|a| \geq 0$ for all $a \in \mathbb{F}$

P-2. $|a| = 0$ if and only if $a = 0$.

P-3. $|a + b| \leq |a| + |b|$.

P-4. $|a \cdot b| = |a| \cdot |b|$.

P-5. Let $a, M \in \mathbb{F}$ and $M \geq 0$. $|a| \leq M$ if and only if $-M \leq a \leq M$.

P-6. $|a - b| \geq |a| - |b|$ for all $a, b \in \mathbb{F}$.

P-7. $|ab| = |a|\,|b|$ for all $a, b \in \mathbb{F}$.

P-8. $|a + b| = |a| + |b|$ if and only if $ab \geq 0$.

P-9. Let $a \in \mathbb{F}$. $|a| < \epsilon$ for all $\epsilon > 0$ if and only if $a = 0$.

P-10. $2|xy| \leq x^2 + y^2$ for all $x, y \in \mathbb{F}$.

P-11. $(ab + cd)^2 \leq (a^2 + c^2)(b^2 + d^2)$ for all $a, b, c, d \in \mathbb{F}$.