

Math 300 – Field Axiom Problems

Calculations. In the following, assume we are working in the field \mathbb{R} , i.e. the real numbers. When solving these problems, show your work step-by-step and list which axiom(s) you are using.

1. Compute $(3 + -5)^{-1} \cdot (1 + 5) + -2 \cdot (2 + 1)^{-1}$.
2. Compute $3 \cdot x + 4 \cdot (x + 2)$ when $x = 1$.
3. Solve $3 \cdot x + 3 = 7$ for x .
4. Compute $(4 + -7) \cdot (-2 + 1)$ by 2 different ways.
5. Factor $x \cdot x + 3 \cdot x + 2$ into two linear terms.

Prove or find a counterexample. Assume all are elements of a field.

1. Define x^2 as $x \cdot x$ and 2 as $1 + 1$, then $(a + b)^2 = a^2 + 2ab + b^2$.
2. Define x^3 as $x^2 \cdot x$ and 3 as $2 + 1$, then $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$.
3. $a \cdot 0 = 0$.
4. $a \cdot b = 0$ if and only if $a = 0$ or $b = 0$.
5. If $a \neq 0$ then $(a^{-1})^{-1} = a$.
6. $-0 = 0$.
7. $-a = -1 \cdot a$.
8. $-(a + -b) = b + -a$.
9. $-1 \cdot -1 = 1$.
10. $(-a) \cdot b = a \cdot (-b) = -(a \cdot b)$.
11. $(-a) \cdot (-b) = a \cdot b$.
12. $1^{-1} = 1$.
13. If $a \neq 0$ then $(-a)^{-1} = -(a^{-1})$.
14. If $a \neq 0$ and $b \neq 0$, then $a^{-1} \cdot b^{-1} = (a \cdot b)^{-1}$.
15. If $a \neq 0$ and $b \neq 0$, then $(a \cdot b^{-1})^{-1} = b \cdot a^{-1}$.
16. If $a \cdot b = a \cdot c$ then $a = 0$ or $b = c$.
17. If $c \neq 0$, then $(a + b) \cdot c^{-1} = a \cdot c^{-1} + b \cdot c^{-1}$.
18. If $a \neq 0$ and $b \neq 0$, then $a^{-1} + b^{-1} = (a + b) \cdot (ab)^{-1}$.
19. If $a \neq 0$ and $b \neq 0$, then $c \cdot a^{-1} + d \cdot b^{-1} = (d \cdot a + c \cdot b) \cdot (ab)^{-1}$.