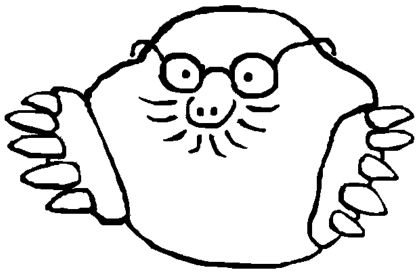

Math Mole

June 25, 2008 *I am the walrus, coo-coo cachu* Volume 3, No. 3

This issue

- Mathematician of the Day
- Quotes
- Puzzles
- Today's Editors: Jim & Dr. C
- Thursday: Jake & Gainer

Quotes: *Son, if you really want something in this life, you have to work for it. Now quiet! They're about to announce the lottery numbers.* – Homer Simpson

Alcohol and math don't mix, so please don't drink and derive. – unknown

Puzzles:

One: Without stacking, what is the largest number of pieces you can make out of one sheet of paper using 3 cuts? How about with n cuts?

Two: Rewrite the following system so that it is much easier to solve:

$6751X + 3249Y = 26751$

$3249X + 6751Y = 23249$

Mathematician of the Day

Agnes Baxter

- Agnes Baxter was born in Halifax, Nova Scotia, Canada. She was a student at Dalhousie University from 1887 to 1892. In 1891 she received her BA with first class honors in Mathematics, the first woman to receive this distinction at Dalhousie, and was the winner of the Sir William Young Gold Medal. In 1892, she received an MA in Mathematics, also from Dalhousie.
- From Dalhousie, Agnes Baxter went to Cornell University where she did graduate work in mathematics, won a fellowship, and was awarded the degree of Ph.D. in 1895. Her thesis, "On Abelian Integrals, a Resume of Neumann's Abelsche Integral with Comments and Applications" was written under the direction of J.E.Oliver. She was the second Canadian woman to receive a Ph.D. in mathematics in North America.
- In 1896 Baxter married A. Ross Hill, also a graduate of Dalhousie with an 1895 Ph.D. in Philosophy from Cornell University. In 1908 Ross Hill became president of the University of Missouri. Unfortunately, Agnes Baxter Hill was in ill health for many years. After her untimely death at the age of 47, President Hill made a gift of books to Dalhousie "... to perpetuate the memory of one of its loyal graduates, who gave her life to assist in my educational work instead of making an independent record for herself." In 1988, Dalhousie dedicated the Agnes Baxter Reading Room in the Department of Mathematics, Statistics and Computing Science.

Taken Directly From: `http://www.agnesscott.edu/lriddle/women/baxter.htm`

Math Spotlight: Fractal Dimensions

What does it mean for a geometric object to have dimension 1? Or dimension 2? How about dimension $3/2$?

Does it make sense to talk about the area of a line or the (total) length of a square?

One way to make sense of dimension is to put all the dimensions under one framework and then identify what dimension you are working in by a number d, with $d \geq 0$. To make this work, you have to figure out how you would measure something in dimension d.

Here we go: Let's assume that $d \leq 3$ and let $S(r)$ be a sphere (in 3-space) of diameter r. We'll define the d-dimensional area (generic word; could be replaced by length or volume as needed) of this sphere to be $C(d)r^d$ where $C(d)$ is independent of r and depends only on the dimension. For example the 1-dimensional area is r ($C(1) = 1$), the 2-dimensional area is πr^2 ($C(2) = \pi$) and the 3-dimensional area is $\frac{4}{3}\pi r^3$ ($C(3) = \frac{4}{3}\pi$).

Now take any geometric object that lives in 3-dimensions. Fix $r > 0$ and cover it with spheres $S(r)$. Use the least number of spheres that you can. We then will approximate the d-dimensional area of your object by simply multiplying the number of spheres you used times the d-dimensional area of $S(r)$. So if it took 123 spheres and you are asking about the $3/2$-dimensional area, you can approximate it by $123 \cdot C(3/2)r^{3/2}$. Next, take r smaller and smaller (but always > 0). As you decrease r you will use more spheres and the approximation of the area will change. In the limit as r goes to 0, we will get a number and that number is the d-dimensional area of your object!

What is the 1-dimensional area of a 1×1 square? Estimating, it would take about $(1/r) \times (1/r)$ spheres $S(r)$ to cover the square. Thus the area is approximately $(1/r)^2 \cdot r = 1/r$. Thus as r goes to 0, the approximation goes to ∞. Does that make sense?

Then what is the 3-dimensional area of the square? By the same estimate the area is approximately $(1/r)^2 C(3)r^3 = C(3)r$. Thus as $r \to 0$, the approximation goes to 0. Does that make sense?

Finally we can figure out what we mean by dimension. Think of the area of an object as we change the dimension d. If d is too small then the area is ∞. If d is too big, then the area is 0. Thus there is one special value d^* such that for $d > d^*$ the area is 0 and for $d < d^*$ the area is ∞. We call d^* the dimension of the object.

Are there objects with fractional dimensions? Yes. Go look up fractals.

Sources: the mind of Dr. C and probably some stuff I've read over the years plus an Analysis course I took in 1985 or so.