Math 300 – Set Theory Problems

For each statement prove it or find a counter-example. A, B and C are subsets of the universe of discourse set U. For any set A we write A^c for $U \setminus A$.

1. $(A \setminus B) \cap B = \emptyset$.
2. $A \cap B = A \setminus (A \setminus B)$.
3. $(A \cup B) \cap (A \cup B^c) = A$
4. $(((A \cap C) \cap B) \cup ((A \cap C) \cap B^c)) \cup (A \cap C)^c = U$
5. $(A \cup C) \cap ((A \cap B) \cup (C^c \cap B)) = A \cap B$
6. $((A \setminus B) \cup (B \setminus A))^c \cap A = A \cap B$.
7. $A \cap B = A \cap (B \setminus A^c)$.
8. $A \setminus B = A \cap B^c$.
9. $(A \cap B) \cup (A \setminus B) = A$.
10. If $x \notin (A \cup B^c) \cup (A \setminus B)$ then $x \in B$.
11. $B \subseteq A$ if and only if $B \setminus A = \emptyset$.
12. $(A \cap B) \cup C = A \cap (B \cup C)$ if and only if $A \subseteq C$.
13. If $A \cap B = A \cap C$ and $A \cup B = A \cup C$ then $B = C$.
14. $A \cap B = B$ if and only if $B \subseteq A$.
15. If $A \subseteq B$ and $B \subseteq C$ and $C \subseteq A$ then $A = B = C$.
16. $B \setminus (B \setminus A) = A$ if and only if $A \subseteq B$.
17. If $A \subseteq B$ then $A \cup B = B$.
18. $A \subseteq C$ if and only if $A \cup (B \cap C) = (A \cup B) \cap C$.
19. $A \triangle \emptyset = A$. (Define: $A \triangle B = (A \setminus B) \cup (B \setminus A)$.)
20. $A \triangle A = \emptyset$.
21. $A \triangle B = B \triangle A$.
22. $A \triangle (B \triangle C) = (A \triangle B) \triangle C$.
23. $A \cap (B \triangle C) = (A \cap B) \triangle (A \cap C)$.
24. $A \triangle B = (A \cup B) \setminus (A \cap B)$.