Field and Order Axioms for $+$, $\cdot$ and $<$ defined on $\mathbf{R}^2$
(*Includes uniqueness results and addition of commutativity)

1. (Closure) For $a, b \in \mathbf{R}$:
   (a) $a + b \in \mathbf{R}$.
   (b) $a \cdot b \in \mathbf{R}$.
2. (Associativity) For $a, b, c \in \mathbf{R}$:
   (a) $a + (b + c) = (a + b) + c$
   (b) $a \cdot (b \cdot c) = (a \cdot b) \cdot c$.
3. (Commutativity) For $a, b, c \in \mathbf{R}$:
   (a) $a + b = b + a$
   (b) $a \cdot b = b \cdot a$.
4. (Distributive Law) For $a, b, c \in \mathbf{R}$:
   $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$.
5. (Additive Identity*) There exists a unique element $0 \in \mathbf{R}$ with
   $a + 0 = 0 + a = a$ for all $a \in \mathbf{R}$.
6. (Multiplicative Identity*) There exists a unique element $1 \in \mathbf{R}$, $1 \neq 0$, with
   $a \cdot 1 = 1 \cdot a = a$ for all $a \in \mathbf{R}$.
7. (Additive Inverse*) For each $a \in \mathbf{R}$ there is a unique element $n(a)$ with
   $a + n(a) = n(a) + a = 0$.
8. (Multiplicative Inverse*) For each $a \in \mathbf{R}$, $a \neq 0$, there is a unique element $q(a)$ with
   $a \cdot q(a) = q(a) \cdot a = 1$.
9. (Trichotomy) If $a, b \in \mathbf{R}$ then one and only one of the following holds: $a < b$, $a = b$ or $a > b$.
10. (Transitive) If $a < b$ and $b < c$ then $a < c$.
11. (Additive) If $a < b$ and $c \in \mathbf{R}$ then $a + c < b + c$.
12. (Multiplicative) if $a < b$ and $c > 0$ then $ac < bc$.
13. (Triangle Inequality) $|x + y| \leq |x| + |y|$ for all x, y.