Math 300 – Set Theory Problems

For each statement prove it or find a counter-example. Unless otherwise stated, A, B and C are subsets of the universal set U.

1. $\{x \in \mathbf{R} : 2 < 5x + 2 \leq 7\} = (0, 1]$.
2. $\{x \in \mathbf{R} : 2 < 1 - x < 3\} = (-2, -1)$.
3. $\{x \in \mathbf{R} : -1 \leq 1 - 2x < 7\} = (-3, 1]$.
4. **Associative Property**. $(A \cup B) \cup C = A \cup (B \cup C)$.
5. **Associative Property**. $(A \cap B) \cap C = A \cap (B \cap C)$.
6. **Commutative Property**. $A \cup B = B \cup A$.
7. **Commutative Property**. $A \cap B = B \cap A$.
8. **Distributive Property**. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
9. **Distributive Property**. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
10. $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$.
11. $A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$.
12. $A \cup \emptyset = A$.
13. $A \cap \emptyset = \emptyset$.
14. $A \cup A = A$.
15. $A \cap A = A$.
16. $A \cup (A \cap B) = A$.
17. $A \cap (A \cup B) = A$.
18. $(A \setminus B) \cap B = \emptyset$.
19. $A \cap B = A \setminus (A \setminus B)$.
20. $(A \cup B) \cap (A \cup B^c) = A$
21. $(((A \cap C) \cap B) \cup ((A \cap C) \cap B^c)) \cup (A \cap C)^c = U$
22. $(A \cup C) \cap ((A \cap B) \cup (C^c \cap B)) = A \cap B$
23. $((A \setminus B) \cup (B \setminus A))^c \cap A = A \cap B$.
24. $B = A \cap (B \cup A^c)$.
25. $A \cap B = A \cap (B \setminus A^c)$.
26. $A \setminus B = A \cap B^c$.
27. $(A \cap B) \cup (A \setminus B) = A$.
28. $A \cup B = (A \setminus B) \cup (B \setminus A)$.

29. If $x \notin (A \cup B^c) \cup (A \setminus B)$ then $x \in B$.

30. $B \subset A$ if and only if $B \setminus A = \emptyset$.

31. $(A \cap B) \cup C = A \cap (B \cup C)$ if and only if $A \subseteq C$.

32. If $A \cap B = A \cap C$ and $A \cup B = A \cup C$ then $B = C$.

33. $A \cap B = B$ if and only if $B \subseteq A$.

34. If $A \subseteq B$ and $B \subseteq C$ and $C \subseteq A$ then $A = B = C$.

35. $B \setminus (B \setminus A) = A$ if and only if $A \subseteq B$.

36. If $A \subseteq B$ then $A \cup B = B$.

37. $A \subseteq C$ if and only if $A \cup (B \cap C) = (A \cup B) \cap C$.

38. $A \bigtriangleup \emptyset = A$. (Define: $A \bigtriangleup B = (A \setminus B) \cup (B \setminus A)$.)

39. $A \bigtriangleup A = \emptyset$.

40. $A \bigtriangleup B = B \bigtriangleup A$.

41. $A \bigtriangleup (B \bigtriangleup C) = (A \bigtriangleup B) \bigtriangleup C$.

42. $A \cap (B \bigtriangleup C) = (A \cap B) \bigtriangleup (A \cap C)$.

43. $A \bigtriangleup B = (A \cup B) \setminus (A \cap B)$.