Matrix Algebra
Test 2 (100pt)

"I ______________________________ will be academically honest is all of my work on this test."
PRINT NAME

X ______________________________
SIGN NAME

No calculators are allowed. Show work for partial credit.

PART I (50pt)

1. Let $B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} \right\}$.

   a. **5pt** Show that B is a basis of $\mathbb{R}^3$.

   b. **10pt** If $\vec{b} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$, find $[\vec{b}]_B$.

2. **3pt** By a familiar name, identify the subspace $gen\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$.

   Answer: ______________________________ matrices in $\mathbb{R}^{2x2}$.

3. Consider $A = \begin{bmatrix} -3 & 2 & -4 & 3 \\ 1 & -2 & 0 & -5 \\ -1 & 1 & -1 & 2 \end{bmatrix}$ and $rref(A) = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

a. **4pt** Find $rank(A)$ and $nullity(A)$.

b. **10pt** Find a basis of $null(A)$.

c. **6pt** Find a basis of $col(A)$.

d. **6pt** Find a basis of $row(A)$.

4. **6pt** Let A be in $\mathbb{R}^{4x6}$. If $rank(A) = 3$, find $rank(A^T)$, $nullity(A)$, and $nullity(A^T)$.

PART II (37pt)

5. **7pt** Let A be in $\mathbb{R}^{5x3}$. The column vectors of A cannot span $\mathbb{R}^5$. Explain.

6. **7pt** Let $V = P_2$, the space of all polynomials of degree less than or equal to 2. The vectors $2x^2 + 3x$, $4x^2 + 1$, $3x - 5$, and $x^2 - x + 1$ cannot be linearly independent in P_2. Explain.

7. **7pt** Let B be in $\mathbb{R}^{6x6}$ and T be a matrix transformation defined by multiplication by B. If B is invertible, then T is both 1-to-1 and onto. Explain.

8. **8pt** Show that $\begin{bmatrix} 1 & 3 \\ 0 & 2 \end{bmatrix}$, $\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$. and $\begin{bmatrix} 5 & 5 \\ 0 & 1 \end{bmatrix}$ form a basis for the upper triangular matrices in $\mathbb{R}^{2x2}$.

9. **8pt** If A is in $\mathbb{R}^{3x3}$, define the trace of matrix A to be $a_{11} + a_{22} + a_{33}$ (the sum of the entries along the diagonal). Let W be the collection of all matrices in $\mathbb{R}^{3x3}$ with a trace of zero. Show that W is a subspace of $\mathbb{R}^{3x3}$.

PART III (13pt)

10. Choose one of the following two problems to complete:

a. **13pt** Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & x+2 & 1 \\ 0 & y-1 & x-2 \end{bmatrix}$ and T be a transformation defined by multiplication by A^T.

Find five integer ordered pairs (x, y) which make T not onto. (Note: an ordered pair (x, y) is an integer ordered pair if both x and y are among the integers: $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$.)

b. **13pt** Let V be a vector space, and let $\vec{u}$, $\vec{v}$ and $\vec{w}$ be nonzero vectors in V. Show that $\vec{u}-\vec{v}$, $\vec{v}-\vec{w}$, and $\vec{w}-\vec{u}$ cannot be linearly independent.