Matrix Algebra
Test 1 (100pt)

"I ______________________________ will be academically honest is all of my work on this test."
PRINT NAME

X ______________________________
SIGN NAME

No calculators are allowed. Show work for partial credit.

PART I (50pt)

1. Let $A = \begin{bmatrix} 1 & 3 & 0 \\ -1 & -2 & 1 \\ 0 & 2 & 1 \end{bmatrix}$.

   a. **15pt** Find A^{-1} (if it exists).

   b. **6pt** If $\vec{b} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$, find all solutions to $A\vec{x} = \vec{b}$.

   c. **4pt** Is A able to be expressed as a product of elementary matrices? (No explanation needed.)

      Circle one: YES NO

2. **12pt** Let $A=\begin{bmatrix}1 & 0 & -1\\ 0 & 2 & 1\end{bmatrix}$, $B=\begin{bmatrix}3 & -2\\ -2 & 1\end{bmatrix}$, and $C=\begin{bmatrix}-1 & 1\\ 1 & 0\end{bmatrix}$, find $A^T B+2A^T C$.

3. **5pt** State the solution to the system: $\left[\begin{array}{ccccc|c}1 & 2 & 0 & 0 & 0 & 5\\ 0 & 0 & 1 & 0 & -4 & -1\\ 0 & 0 & 0 & 1 & 0 & 3\end{array}\right]$.

4. **4pt** If $T:\mathbb{R}^3 \rightarrow \mathbb{R}^2$ such that $T(\vec{x})=\begin{bmatrix}4x_1-3x_2\\ x_1+x_2-x_3\end{bmatrix}$, find the standard matrix which represents T.

5. **4pt** If $\vec{a}=\begin{bmatrix}1\\ -2\\ 4\\ 2\end{bmatrix}$, find $\|\vec{a}\|$.

PART II (37pt)

6. **7pt** The system $\left[\begin{array}{cccc|c} * & * & * & * & * \\ * & * & * & * & * \\ 0 & 0 & 0 & 0 & 4 \end{array}\right]$ has no solutions, regardless of the values in r_1 and r_2. Explain.

7. **7pt** A is a 4×4 matrix that is both symmetric and upper triangular. If $a_{22} = 0$, then $rref(A) \neq I$. Explain.

8. **7pt** If $\vec{a} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} c \\ 5 \\ c \end{bmatrix}$, then regardless of the value of c, $\vec{a}$ and $\vec{b}$ are not orthogonal. Explain.

9. **8pt** If $E = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $EB = \begin{bmatrix} 0 & -2 & 0 & 1 \\ -2 & 1 & 1 & 5 \\ 0 & -1 & 0 & 0 \\ 1 & 1 & -2 & -3 \end{bmatrix}$, find $|B|$.

10. **8pt** If $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$, then $A\vec{x} = \vec{b}$ has either no solutions or infinitely many solutions. Explain.

PART III (13pt)

11. Choose one of the following two problems to complete:

a. **13pt** Let A be a 3×3 diagonal matrix such that $A^2-3A-4I$ equals the zero matrix. Find all possible values for $|A|$.

b. **13pt** Let A be a non-square matrix (ie. Thm 1 does not apply). Let $\vec{u}$ be a solution to $A\vec{x}=\vec{b}$. Let $\vec{v}$ be a solution to $A\vec{x}=\vec{0}$.
   (i) Show that $\vec{u}+c\vec{v}$ is a solution to $A\vec{x}=\vec{b}$ for every scalar c.
   (ii) Use (i) to explain why if $\vec{v}\neq\vec{0}$, then $A\vec{x}=\vec{b}$ must have infinitely many solutions.