

### Test 3 Skills Checklist

- Know what is meant by an eigenvalue ( $c$ ) and eigenvector ( $\vec{v} \neq \vec{0}$ ) of an  $n \times n$  matrix  $A$  ( $A\vec{v} = c\vec{v}$ ).
- Given an  $A$  in  $\mathbb{R}^{n \times n}$ , be able to find the eigenvalues ( $|A - cI| = 0$ ) and eigenvectors (solve  $[A - cI \mid 0]$ ).
- Know that each eigenvalue has at least one eigenvector ( $|A - cI| = 0$  means  $\text{nullity}(A - cI) > 0$ ).
- Know that eigenvectors are linearly independent, and hence are at most  $n$  in number for a given  $A$ .
- Know that the value of  $\text{nullity}(A - cI)$  is the number of eigenvectors associated with the eigenvalue  $c$ .
- Know the term eigenspace refers to the subspace generated by all eigenvectors (ie.  $\text{gen}\{\vec{v}_1, \dots, \vec{v}_n\}$ ).
- Know what is meant by " $A$  is similar to  $B$ ", namely that  $P^{-1}A = BP^{-1}$ .
- Know that, in effect, if  $A$  is similar to  $B$ , then they're the same transformation under a change of basis.
- Know Thm 4 (TFAE:  $A$  is similar to a diagonal matrix  $D$ ;  $A$  has  $n$  eigenvectors).
- Know the mechanism of theorem 4, namely  $P = [\vec{v}_1 \mid \dots \mid \vec{v}_n]$ ,  $D = \begin{bmatrix} c_1 & & \\ & \ddots & \\ & & c_n \end{bmatrix}$ , and  $P^{-1}A = DP^{-1}$ .
- Be able to use Thm 4 to compute large powers of  $A$  without multiplying  $A$  by itself.
- Know that a rigid transformation  $T$  preserves length ( $\|\vec{x}\| = \|T(\vec{x})\|$ ) and angle ( $\vec{x} \cdot \vec{y} = T(\vec{x}) \cdot T(\vec{y})$ ).
- Know that an orthonormal basis contains only normal ( $\|\vec{a}_i\| = 1$ ) and orthogonal ( $\vec{a}_i \cdot \vec{a}_j = 0$ ) vectors.
- Know the fact that  $n \times n$  matrices where  $A^{-1} = A^T$  link rigid transformations to orthonormal bases.
- Know the fact that matrices with orthonormal eigenspaces are necessarily symmetric.