Analysis Diagnostic Exam August 12, 2019

NAME: ______________________________

#1.) ____/15 #2.) ____/20 #3.) ____/15 #4.) ____/15 #5.) ____/15 #6.) ____/20

Total: ______/100

Instructions: There are 100 points possible on this exam. If you have any question about the notation or meaning of any question, please ask the exam proctor. You must show all necessary steps to get full credit. Partial credit will only be given for progress toward a correct solution.

1.) (15 points) Suppose A is a set, and B_λ is a set, for each λ belonging to some index set Λ. Prove that

$$A \cap (\cup_{\lambda\in\Lambda} B_\lambda) = \cup_{\lambda\in\Lambda}(A \cap B_\lambda).$$

2.) (20 points) Prove directly from the definition of the limit of a sequence (that is, without using any limit theorems) that

$$\lim_{n\to\infty} \frac{3n^2+1}{2n^2+7} = \frac{3}{2}.$$

3.) (15 points) Let $A \subseteq \mathbb{R}$ be non-empty, and let $y \in \mathbb{R}$. Define the translate $A + y$ of A by

$$A + y = \{a + y : a \in A\}.$$

If A is open, prove that $A + y$ is open.

4.) (15 points) Suppose f and g are real-valued functions on $\mathbb{R}$, and c is a point in $\mathbb{R}$. Suppose

$$\lim_{x \to c} f(x) = 0,$$

and g is bounded (that is, there exists $M > 0$ such that $|g(x)| \leq M$ for all $x \in \mathbb{R}$). Prove that

$$\lim_{x \to c} (f(x)g(x)) = 0.$$

5.) (15 points) Using only the open cover definition of compactness (and not the Heine-Borel or Bolzano-Weierstrass theorems), prove that if $K \subseteq \mathbb{R}$ is compact, then K is bounded (that is, there exists $M > 0$ such that $|x| \leq M$ for all $x \in K$).

6.) (20 points) Suppose $a, b, c \in \mathbb{R}$ with $a < c < b$. Suppose $f : [a, b] \to \mathbb{R}$ is bounded. If f is Riemann integrable on $[a, b]$, prove that f is Riemann integrable on $[a, c]$.