

2002 Pro2Serve Tennessee Math Contest

Fermat I

1. Simplify: $2x^2y \sqrt[3]{2x} + 7x^2 \sqrt[3]{2xy^3} - 4 \sqrt[3]{16x^7y^3}$

(a) $x^6y^3 \sqrt[3]{2x}$

(d) $2x^3y$

(b) $x^2y \sqrt[3]{2x}$

(c) $9x^2y \sqrt[3]{2x} - 8y \sqrt[3]{2x^7y}$

(e) None of the above

2. Write as a sum, difference, or multiple of logarithms: $\log \sqrt[3]{\frac{a^2b}{c}}$

(a) $\sqrt[3]{\frac{2 \log a + \log b}{\log c}}$

(c) $\frac{1}{3}(2 \log a + \log b - \log c)$

(d) $\sqrt[3]{2 \log a^2 + \log b - \log c}$

(b) $\frac{1}{3} \left(\frac{2 \log a + \log b}{\log c} \right)$

(e) None of the above

3. Find the partial fraction decomposition of $\frac{6x - 13}{x^2 - 2x + 1}$

(a) $\frac{6x}{x - 1} + \frac{13}{(x - 1)^2}$

(c) $\frac{6}{x - 1} - \frac{6x + 7}{(x - 1)^2}$

(d) $\frac{4}{x - 1} + \frac{2}{(x - 1)^2}$

(b) $\frac{6}{x - 1} - \frac{7}{(x - 1)^2}$

(e) None of the above

4. Order the following numbers from smallest to largest: $x = 2^{1999}$, $y = 1999^2$, $z = 10^{\log_2 1999}$.

(a) y, x, z

(d) x, y, z

(b) y, z, x

(e) None of the above

(c) z, y, x

5. In the expansion of $(a + b + c)^6$, what is the coefficient of a^4bc ?

- (a) 10 (d) 25
(b) 15 (e) None of the above
(c) 20

6. The decimal equivalent of the binary number 10110111 is

- (a) 152 (d) 205
(b) 178 (e) None of the above
(c) 183

7. Find the positive number d such that $x^2 + (d - 3)x + 2d + 6 = 0$ has one real root.

- (a) $d = 1$ (d) $d = 15$
(b) $d = 5$ (e) None of the above
(c) $d = 11$

8. For which value of a does the straight line $y = 6x$ intersect the parabola $y = x^2 + a$ in exactly one point?

- (a) 9 (d) 11
(b) 10 (e) None of the above
(c) 8

9. The equation of a line passing through the point $(2, 3)$ and perpendicular to the given line $5x - 2y = 10$ is

- (a) $y = \frac{5}{2}x - 2$ (d) $y = -\frac{2}{5}x + \frac{19}{5}$
(b) $y = -\frac{5}{2}x + 8$
(c) $y = \frac{2}{5}x + \frac{11}{5}$ (e) None of the above

10. For what value a the following linear system has more than *one* solution

$$\begin{cases} ax + y + z = 0 \\ 2y - z = 0 \\ x + 3y = 0 \end{cases}$$

- (a) $a = -1$ (d) $a = \frac{8}{3}$
(b) $a = \frac{3}{4}$ (e) None of the above
(c) $a = 1$

11. Let $y_1 = -x^2 + 4x - 2$ and $y_2 = x - 2$. For what values of x will $y_1 \geq y_2$?

- (a) $0 \leq x \leq 3$ (d) $-3 \leq x \leq 0$
(b) $-1 \leq x \leq 0$ (e) None of the above
(c) $1 \leq x \leq 5$

12. Find the value of $x = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{\dots}}}$, accurate to the fourth decimal place.

- (a) 1.5564 (d) 1.7321
(b) 1.6180 (e) None of the above
(c) 1.4142

13. A book has 500 pages numbered 1, 2, 3, \dots , and so on. How many times does the digit 1 appear in the page numbers ?

- (a) 100 (d) 250
(b) 150
(c) 200 (e) None of the above

14. If 3 dice are thrown, in how many ways can we obtain a sum of 15? (A toss of 6, 6, 3, is different from a toss of 6, 3, 6).

- (a) 5 (d) 12
(b) 7
(c) 10 (e) None of the above

15. What is the probability, to the nearest percent, that of any seven persons chosen at random, exactly one was born on a Tuesday ?

- (a) 40% (d) 32%
(b) 14%
(c) 20% (e) None of the above

16. If you have a test with 6 questions and 4 possible answers to each question, and you guess all answers, what is the probability of getting all 6 questions right?

(a) $\frac{18}{4096}$

(d) $\frac{1}{4096}$

(b) $\frac{12}{4096}$

(e) None of the above

(c) $\frac{6}{4096}$

17. The area of a triangle having vertices $(1, 0)$, $(4, 4)$ and $(-3, 2)$ is

(a) 11 square units

(d) 13 square units

(b) 10 square units

(e) None of the above

(c) 4 square units

18. Find center of the hyperbola: $3x^2 - 4y^2 - 6x - 16y + 7 = 0$

(a) $(4, 3)$

(d) $(3, -8)$

(b) $(1, -2)$

(c) $(1, -8)$

(e) None of the above

19. In the following diagram (Figure 1) of the front view of the Great Pyramid, the measure of the angle PRO is 120 degrees, and the measure of the angle PST is 110 degrees. What is the measure of the angle RPS in degrees?

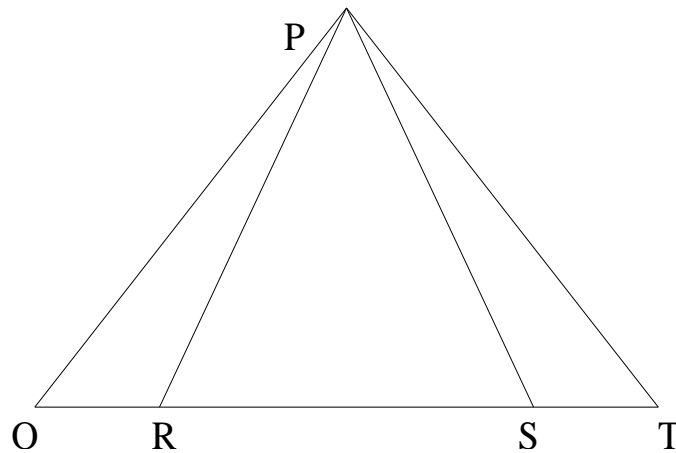


Figure 1: the Great Pyramid

(a) 65 degrees

(d) 50 degrees

(b) 60 degrees

(c) 55 degrees

(e) None of the above

- (a) 593.28 m^2
- (b) 489.32 m^2
- (c) 1600 m^2
- (d) 1885.12 m^2
- (e) None of the above

22. If the side of the outer square in Figure 4 has a length of 5 inches, find the sum of the perimeters of all three squares in the figure (accurate to the fourth decimal place).

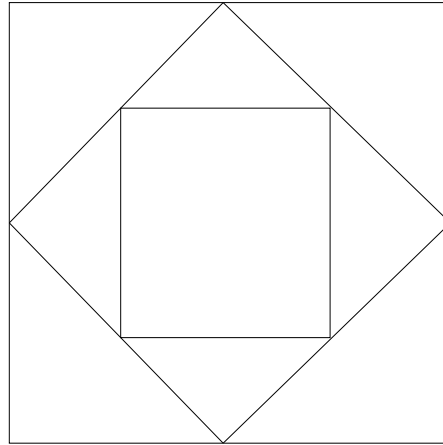


Figure 4: Squares inside a square

- (a) 60 inches
- (b) 55.1024 inches
- (c) 44.1421 inches
- (d) 38 inches
- (e) None of the above

23. A ladder leans against a vertical wall. The top of the ladder is 7 meters above the ground. When the bottom of the ladder is moved 1 meter farther away from the wall, the top of the ladder rests against the foot of the wall. What is the length of the ladder?

- (a) 25 meters
- (b) 20 meters
- (c) 14 meters
- (d) 8 meters
- (e) None of the above

24. How many whole years will be needed for \$5000 to increase to \$22,000 at 11 percent interest compounded monthly?

- (a) 19 years
- (b) 14 years
- (c) 9 years
- (d) 5 years
- (e) None of the above

25. John has an income which is five eighths that of Peter. John's expenses are one-half those of Peter and John saved 40% of his income. What is the percentage of his income that Peter saves?

(a) 15%

(b) 20%

(c) 25%

(d) 30%

(e) None of the above