

1999 Probe I

- 1) If $f(x) = x^4 - 1$, then $f\left(\frac{c}{b}\right)$, $b \neq 0$, equals
- a) $\frac{c^2 - b^2}{b^4}$
 - b) $\frac{c^4 - 1}{b^4}$
 - c) $\frac{c^4 - b^4}{b^4}$
 - d) $\frac{c^4 - 1}{b^4 - 1}$
 - e) $\frac{c^4}{b^4 - 1} - 1$
- 2) If the numbers $x = 3^{36}$, $y = 9^{20}$, $z = 27^8$ were arranged in decreasing numerical order, the ordering would be
- a) $z > y > x$
 - b) $x > y > z$
 - c) $y > x > z$
 - d) $z > x > y$
 - e) $y > z > x$
- 3) $-1 - (-1)^5 =$
- a) -2
 - b) -1
 - c) 0
 - d) 1
 - e) 2
- 4) If the length of a rectangle is halved and the width is tripled, the area of the rectangle has been multiplied by
- a) $\frac{2}{3}$
 - b) 6
 - c) $1\frac{1}{2}$
 - d) $\frac{1}{6}$
 - e) $3\frac{1}{2}$

- 5) A playground has 3 entrances, each equally likely to be used. What is the probability that two children entering the playground will use the same entrance?
- a) $\frac{1}{3}$
 - b) $\frac{1}{2}$
 - c) $\frac{2}{3}$
 - d) $\frac{1}{6}$
 - e) $\frac{1}{9}$
- 6) A mathematics test has 25 questions. Four points are given for each correct answer 1 point is deducted for each incorrect answer. If Brad answered all questions and scored 35, then the number of questions he answered *incorrectly* was
- a) 12
 - b) 10
 - c) 13
 - d) 15
 - e) 20
- 7) The yearly change in the yield of corn over four consecutive years was as follows: An increase of X% in each of the first two years, followed by a decrease of X% in each of the next two years. Assuming that at least some corn was produced during the period and $0 < X < 100$, what has happened to the yield of corn from the beginning of the four-year period to the end?
- a) increased
 - b) decreased
 - c) unchanged
 - d) dropped to 0
 - e) unable to tell from the given information
- 8) If $n \geq 2$ is a natural number, which of the following integers *must* be divisible by 3?
- a) $n(n^2 - 1)$
 - b) $n^2(n+1)$
 - c) $n^2 + 3n + 2$
 - d) all of the above
 - e) none of the above

9) If $a - b = 5$, $b + c = 2$, $c - a = 8$, then $\frac{b}{a} =$

- a) $-\frac{2}{11}$
- b) 11
- c) $\frac{5}{8}$
- d) $\frac{1}{4}$
- e) $\frac{2}{5}$

10) If the radius of a circle is decreased from 5 inches to 3 inches, by what percentage is its area decreased?

- a) 36%
- b) 40%
- c) 60%
- d) 64%
- e) 75%

11) If $\sqrt{x} + \sqrt{x} = \sqrt{24}$, then x is equal to

- a) 12
- b) 6
- c) $\sqrt{12}$
- d) $\sqrt{6}$
- e) $\sqrt{3}$

12) Two consecutive odd positive integers have a product equal to 255. What is their sum?

- a) 15
- b) 35
- c) 32
- d) 42
- e) 46

13) An ant walks 3 feet, then changes direction by 18° to the left, walks another 3 feet, etc. If the ant walks at the rate of 1 foot per 15 minutes, how long does it take for the ant to return to its starting point?

- a) 60 hours
- b) 20 hours
- c) 15 hours
- d) 18 hours
- e) forever

- 14) How many numbers are in the following list of numbers: 2, 6, 10, 14, 18, ..., 186?
- a) 184
 - b) 182
 - c) 46
 - d) 48
 - e) 47
- 15) Chelsea owns 2 different skirts, 5 different blouses, and 3 different pairs of slacks. The only combination she will not wear is her orange blouse with her pink skirt. Beginning on the first day of school, she decides to wear a different outfit on each school day as long as possible. On what day must she finally wear an outfit that she has already worn?
- a) 25th
 - b) 24th
 - c) 26th
 - d) 29th
 - e) 30th
- 16) If $x^y = \frac{1}{2}$ then $(x^2)^{3y} =$
- a) $\frac{1}{8}$
 - b) $\frac{1}{32}$
 - c) 8
 - d) $\frac{1}{64}$
 - e) 32
- 17) If $(x + 3)(3x - 5) = 0$, then the possible values of $3x - 5$ are:
- a) $\frac{5}{3}$ and - 3
 - b) 0 only
 - c) 0 and - 14
 - d) 0 and 4
 - e) $\frac{5}{3}$ and - 14

18) If $f(x) = \frac{1}{\frac{1}{x} + 1}$, then $\frac{f(x)}{f\left(\frac{1}{x}\right)} =$

- a) 1
- b) x
- c) $\frac{1}{x}$
- d) x^2
- e) $\frac{1}{\frac{1}{x^2} + 1}$

19) If $(-2, 16)$ is the minimum point on the graph of $y = 3x^2 + 4kx - m$, where k and m are constants, then m equals

- a) -28
- b) -4
- c) 20
- d) 28
- e) 4

20) When the polynomial $x^4 + x$ is divided by the polynomial $x - 5$, the remainder is

- a) 620
- b) 625
- c) 630
- d) $x^3 - 630$
- e) $x - 635$

21) A pile of gold dust is divided among three prospectors. Crazy Jane Moore gets $\frac{2}{5}$ of the dust, Wild Lena Olson gets $\frac{1}{4}$ of the dust, and Billy "Gold Vein" Smith gets the remaining 14 grams.

How many grams does Wild Lena get?

- a) $\frac{7}{2}$
- b) $\frac{20}{3}$
- c) 20
- d) 17
- e) 10

- 22) What is the largest number of points in which the graphs of a third degree polynomial and a fifth degree polynomial can meet?
- a) 3
 - b) 5
 - c) 8
 - d) 2
 - e) 15
- 23) Jill is twice as old as her sister. Two years ago she was four times as old as her sister. What is the difference between their ages, in years?
- a) 2
 - b) 3
 - c) 5
 - d) 1
 - e) 4
- 24) John has four girlfriends, none of whom knows about the others. Two of the girls are both named Brenda, and the other two are named Holly and Joy. He writes each girl a love letter using her first name, but after sealing the notes in the envelopes he realizes that he has forgotten to write any of the girls' names on the outside of the envelopes. John is unable to tell which letter goes to which girl. But John thinks he is a lucky guy, and, except for the names, the letters are all the same anyway. So he just gives one letter to each girl. What is the probability that, upon opening the letters, each girl sees her own name?
- a) $\frac{1}{24}$
 - b) $\frac{1}{4}$
 - c) $\frac{1}{12}$
 - d) $\frac{1}{16}$
 - e) $\frac{1}{3}$
- 25) Melissa can row her rowboat at 4 mile per hour. She keeps her boat pointed directly towards the opposite bank of a river, and rows across, but as she rows, the current carries her downstream. In order to return from the opposite bank to her starting point across the river, she must row in a direction 60° from the direction of the riverbank. How fast is the river flowing?
- a) 8 mph
 - b) 2 mph
 - c) 6 mph
 - d) $\frac{4\sqrt{3}}{3}$ mph
 - e) $\frac{\sqrt{3}}{4}$ mph