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Linear operators in local fields of prime characteristic

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1. Introduction

In 1958 Mahler [5] proved a strong Weierstrass approximation theorem for the field of p-adic numbers by showing that every continuous p-adic function f on the valuation ring of Q_p is the uniform limit of an interpolation series

$$f(t) = \sum_{n=0}^{\infty} A_n \binom{t}{n}, \tag{1.1}$$

where the coefficients in (1.1) are uniquely determined by

$$A_n = \Delta^n f(0) = \sum_{k=0}^{n} (-1)^k \binom{n}{k} f(n-k). \tag{1.2}$$

In addition, Mahler stated necessary and sufficient conditions involving the coefficients A_n for such a function to be differentiable at a given point.

In the present paper, we prove analogues of Mahler's theorems for continuous linear operators defined in local fields of prime characteristic, i. e., in fields of formal power series over finite fields.

Let $GF((q, x))$ denote the field of formal power series over the finite field $GF(q)$, and let $GF[[q, x]]$ denote the valuation ring of $GF((q, x))$ for the usual absolute value. [See section 3.] We show (Theorem 4. 2) that every continuous linear operator f on the $GF(q)$-vector space $GF[[q, x]]$ is the uniform limit of an interpolation series

$$f(t) = \sum_{n=0}^{\infty} \Delta^n f(1) \frac{\Psi_n(t)}{F_n}, \tag{1.3}$$

where the operators Δ^n are defined recursively by

$$\begin{aligned} \Delta^0 f(t) &= f(t) \\ \Delta^1 f(t) &= f(xt) - x f(t) \\ \Delta^{n+1} f(t) &= \Delta^n f(xt) - x^{q^n} \Delta^n f(t), \end{aligned} \tag{1.4}$$

$$\Psi_n(t) = \prod_{\deg m < n} (t - m) \quad (m \in GF[q, x]), \tag{1.5}$$

and F_n is the product of all monic polynomials in $GF[q, x]$ of degree n. The proof is patterned on Mahler's argument and makes extensive use of interpolation properties of the polynomials $\Psi_n(t)$ due to Carlitz [2].

In addition, we prove (Theorems 5. 1 and 5. 2) that the continuous function f given by (1. 3) is everywhere differentiable on $GF[[q, x]]$ if and only if

$$\lim_{n\to\infty} \frac{\Delta^n f(1)}{L_n} = 0, \tag{1. 6}$$

where L_n is the l. c. m. of all polynomials in $GF[q, x]$ of degree n; and that, if (1. 6) holds, then

$$f'(u) = \sum_{n=0}^{\infty} (-1)^n \frac{\Delta^n f(1)}{L_n} \tag{1. 7}$$

for all $u \in GF[[q, x]]$. It follows that

$$f(t) = \sum_{n=0}^{\infty} x^n \frac{\Psi_n(t)}{F_n} \tag{1. 8}$$

is an example of a continuous nowhere differentiable linear operator on $GF[[q, x]]$.

2. Preliminaries

Let $GF[q, x]$ be the ring of polynomials over the finite field $GF(q)$, and let $GF(q, x)$ be the quotient field of $GF[q, x]$. Define a sequence of polynomials $\Psi_r(t)$ over $GF[q, x]$ by

$$\Psi_r(t) = \prod_{\deg m < r} (t - m), \quad \Psi_0(t) = t \tag{2. 1}$$

where the product in (2. 1) extends over all polynomials $m \in GF[q, x]$ (including o) of degree $< r$. It follows that

$$\Psi_r(t) = \sum_{i=0}^{r} (-1)^{r-i} \begin{bmatrix} r \\ i \end{bmatrix} t^{q^i}, \tag{2. 2}$$

where

$$\begin{bmatrix} r \\ i \end{bmatrix} = \frac{F_r}{F_i L_{r-i}^{q^i}}, \quad \begin{bmatrix} r \\ 0 \end{bmatrix} = \frac{F_r}{L_r}, \quad \begin{bmatrix} r \\ r \end{bmatrix} = 1 \tag{2. 3}$$

and

$$\begin{aligned} F_r &= [r][r-1]^q \cdots [1]^{q^{r-1}}, \quad & F_0 = 1 \\ L_r &= [r][r-1] \cdots [1], & L_0 = 1 \\ [r] &= x^{q^r} - x. \end{aligned} \tag{2. 4}$$

Note that $\Psi_r(x^r) = \Psi_r(m)$ for m monic of degree r, so that F_r is the product of all monic polynomials in $GF[q, x]$ of degree r. On the other hand, L_r may be seen to be the l. c. m. of all polynomials in $GF[q, x]$ of degree r (cf. [3]).

Let K be any extension field of $GF(q, x)$. By (2. 2) the polynomial functions associated to the polynomials $\Psi_r(t)$ are linear operators on the $GF(q)$-vector space K. It is easily seen that a polynomial $f(t) \in K[t]$ whose associated polynomial function is a linear operator on the $GF(q)$-vector space K has the form

$$f(t) = \sum_{i=0}^{n} a_i t^{q^i} \; (a_i \in K). \tag{2. 5}$$

Moreover, the sequence $\{\Psi_r(t)\}_{r\geq 0}$ is an ordered basis of the K-vector space of such "linear" polynomials. Indeed, if $f(t)$ has the form (2. 5), then

$$f(t) = \sum_{i=0}^{n} \Delta^i f(1) \frac{\Psi_i(t)}{F_i}, \tag{2. 6}$$

where the operators Δ^i are defined recursively by

$$
\begin{aligned}
&\Delta^0 f(t) = f(t) \\
(2.7) \qquad &\Delta^1 f(t) = \Delta f(t) = f(xt) - xf(t) \\
&\Delta^{i+1} f(t) = \Delta^i f(xt) - x^{q^i} \Delta^i f(t) \quad [2].
\end{aligned}
$$

It is useful to generalize the polynomials $\Psi_r(t)$ to a set of interpolation polynomials for the full K-vector space $K[t]$. Let r be a positive integer, and write

$$(2.8) \qquad r = e_0 + e_1 q + \cdots + e_s q^s \quad (0 \leqq e_i < q).$$

Define polynomials $G_r(t)$ and $G_r^*(t)$ over $GF[q, x]$ by

$$(2.9) \qquad G_r(t) = \Psi_0^{e_0}(t) \cdots \Psi_s^{e_s}(t), \; G_0(t) = 1$$

and

$$(2.10) \qquad G_r^*(t) = \prod_{i=0}^{s} G_{e_i q^i}^*(t),$$

where

$$(2.11) \qquad G_{eq^i}^* = \begin{cases} \Psi_i^e(t) \text{ for } 0 \leqq e < q-1 \\ \Psi_i^e(t) - F_i^e \text{ for } e = q-1. \end{cases}$$

In particular,

$$(2.12) \qquad G_{q^r-1}^*(t) = \frac{\Psi_r(t)}{t}.$$

Evidently the sequences $\{G_r(t)\}_{r \geqq 0}$ and $\{G_r^*(t)\}_{r \geqq 0}$ are ordered bases of the K-vector space $K[t]$.

Finally, we shall find it useful to employ generalizations of the polynomials F_r. If r is given by (2.8), set

$$(2.13) \qquad g_r = F_1^{e_1} \cdots F_s^{e_s}, \; g_0 = 1.$$

(The polynomials g_r may be regarded as analogues of the integers $r!$.) For further details on the above, the reader is referred to Carlitz [2], [4].

3. The field of formal power series over $GF(q)$

Let $GF((q, x))$ denote the field of formal power series over $GF(q)$. If

$$\alpha \in GF((q, x)) - \{0\},$$

and

$$(3.1) \qquad \alpha = \sum_{i=-\infty}^{\infty} a_i x^i,$$

where all but a finite number of the a_i vanish for $i < 0$, set $v(\alpha) = n$, where n is the least integer for which $a_n \neq 0$. Fix a real number b such that $0 < b < 1$ and set $|\alpha| = b^{v(\alpha)}$. Then $|\,|$ is a discrete non-archimedean absolute value on $GF((q, x))$ and $GF((q, x))$ is complete with respect to $|\,|$. The valuation ring of $GF((q, x))$, denoted by $GF[[q, x]]$, consists of the ring of formal power series of the form

$$(3.2) \qquad \alpha = \sum_{i=0}^{\infty} a_i x^i.$$

$GF[q, x]$ is dense in $GF[[q, x]]$, as is $GF(q, x)$ in $GF((q, x))$. Furthermore, $GF[[q, x]]$ is open and compact, so that $GF((q, x))$ is locally compact. Indeed, it is known that every locally compact field of prime characteristic is topologically isomorphic to some power series field $GF((q, x))$ ([7], pp. 10—12).

To conclude this section, we note a useful bound on the polynomial functions Ψ_r, G_r, and G_r^*, restricted to $GF[[q, x]]$. It is shown in [4] that if $m \in GF[q, x]$, then $\Psi_r(m)/F_r \in GF[q, x]$, $G_r(m)/g_r \in GF[q, x]$, and $G_r^*(m)/g_r \in GF[q, x]$. Since $GF[q, x]$ is dense in $GF[[q, x]]$ and the polynomial functions associated to the polynomials $\Psi_r(t)/F_r$, $G_r(t)/g_r$, and $G_r^*(t)/g_r$ are obviously continuous, it follows that if $\alpha \in GF((q, x))$ and $|\alpha| \leqq 1$ then $|\Psi_r(\alpha)/F_r| \leqq 1$, $|G_r(\alpha)/g_r| \leqq 1$, and $|G_r^*(\alpha)/g_r| \leqq 1$. In fact, it can be shown that $\{\Psi_r(t)/F_r\}_{r\geqq 0}$ is an ordered basis of the $GF[[q, x]]$-module of "linear" polynomials over $GF((q, x))$ which map $GF[[q, x]]$ into itself, and that $\{G_r(t)/g_r\}_{r\geqq 0}$ and $\{G_r^*(t)/g_r\}_{r\geqq 0}$ are ordered bases of the $GF[[q, x]]$-module of polynomials over $GF((q, x))$ which map $GF[[q, x]]$ into itself [6].

4. Interpolation series for continuous linear operators

In [6] it is shown that every continuous function $f\colon GF[[q, x]] \to GF[[q, x]]$ is the uniform limit of a unique interpolation series

$$f(t) = \sum_{i=0}^{\infty} A_i \frac{G_i(t)}{g_i}, \tag{4.1}$$

where, for any r such that $i < q^r$,

$$A_i = (-1)^r \sum_{\deg m < r} f(m) \frac{G^*_{q^r-1-i}(m)}{g_{q^r-1-i}} \quad (m \in GF[q, x]). \tag{4.2}$$

It is also proved as a corollary to (4.1) that such a function f is a linear operator on the $GF(q)$-vector space $GF[[q, x]]$ precisely when the coefficients A_i vanish for i not a power of q, in which case

$$f(t) = \sum_{i=0}^{\infty} B_i \frac{\Psi_i(t)}{F_i} \quad (B_i = A_{q^i}). \tag{4.3}$$

The proof of (4.1) employs a set of auxiliary polynomials $Q_i(t)$ along with theorems of Amice [1] concerning the use of these polynomials in constructing interpolation series for continuous functions. While a more self-contained proof of (4.1) would certainly be desirable, we have not yet succeeded in constructing such a proof (although this would follow from a proof that $\{A_i\}$ is a null sequence, where A_i is given by (4.2)). If we restrict our attention to linear operators, however, the coefficients B_i of (4.3) may be exhibited in a more tractable form, and a self-contained proof of (4.3) may be given as follows:

Lemma 4.1. *For all* $i \geqq 0$

$$\Psi_i\left(\frac{1}{x+1}\right) = \frac{(-1)^i F_i}{(x+1)^e} \quad (e = 1 + q + q^2 + \cdots + q^i). \tag{4.4}$$

Proof. The recurrence

$$\Psi_i(t) = \Psi_{i-1}^q(t) - F_{i-1}^{q-1}\Psi_{i-1}(t) \quad [2] \tag{4.5}$$

may be used to give an inductive proof of (4. 4). Assuming (4. 4) as hypothesis, one has

$$(4.6)\quad \Psi_{i+1}\left(\frac{1}{x+1}\right) = (-1)^i \frac{F_i^q}{(x+1)^{q+q^2+\cdots+q^{i+1}}} - (-1)^i \frac{F_i^{q-1}F_i}{(x+1)^{1+q+\cdots+q^i}}$$

$$= (-1)^{i+1} \frac{F_i^q(x^{q^{i+1}}-x)}{(x+1)^{1+q+\cdots+q^{i+1}}}$$

$$= (-1)^{i+1} \frac{F_{i+1}}{(x+1)^{1+q+\cdots+q^{i+1}}},$$

from which the desired result follows.

Theorem 4. 1. *Let $\{A_i\}_{i\geqq 0}$ be a sequence in $GF((q, x))$. Then the series*

$$(4.7)\qquad \sum_{i=0}^{\infty} A_i \frac{\Psi_i(t)}{F_i}$$

converges uniformly for all $t \in GF[[q, x]]$ if and only if $\{A_i\}$ is null.

Proof. Sufficiency. If $t \in GF[[q, x]]$ then, by a previous remark, $|\Psi_i(t)/F_i| \leqq 1$. Hence, if $\{A_i\}$ is null, $\{A_i\Psi_i(t)/F_i\}$ is null and (4. 7) converges uniformly as $|\,|$ is non-archimedean.

Necessity. By Lemma 4. 1,

$$(4.8)\qquad \left|\frac{\Psi_i\left(\frac{1}{x+1}\right)}{F_i}\right| = \left|\frac{(-1)^i}{(x+1)^{1+q+\cdots+q^i}}\right| = 1$$

for all i. Hence $\{A_i\}$ must be null if (4. 7) is to converge for $t = 1/x+1$.

Theorem 4. 2. *Let f be a continuous linear operator on the $GF(q)$-vector space $GF[[q, x]]$. Then the series*

$$(4.9)\qquad \sum_{i=0}^{\infty} \Delta^i f(1) \frac{\Psi_i(t)}{F_i},$$

where the operators Δ^i are defined by (1. 4), converges uniformly to f on $GF[[q, x]]$. Moreover, the coefficients in (4. 9) are uniquely determined by f.

Proof. It follows from (1. 4) that

$$(4.10)\qquad \Delta^i f(1) = C_0 f(1) + C_1 f(x) + \cdots + C_i f(x^i),$$

where

$$(4.11)\qquad \begin{aligned} C_i &= 1 \\ C_j &= (-1)^{i-j} \sum_{e\in S_j} x^e \quad (0 \leqq j < i), \end{aligned}$$

S_j being the set of all sums of distinct elements of $\{1, q, \ldots, q^{i-1}\}$ taken $i-j$ at a time. Hence $v(C_i) = 0$ and $v(C_j) = 1 + q + \cdots + q^{i-j-1}$, where $0 \leqq j < i$. Given any positive integer R, there exists, by continuity of f at 0, a positive integer N_1 such that $j \geqq N_1$ implies that $v(f(x^j)) \geqq R$. There exists also a positive integer N_2 such that $k \geqq N_2$ implies that $1 + q + \cdots + q^k \geqq R$. Let $N = N_1 + N_2$. Then if $i \geqq N$, it follows from (4. 10) and (4. 11) that $v(\Delta^i f(1)) \geqq R$. Hence $\{\Delta^i f(1)\}_{i\geqq 0}$ is a null sequence, and by Theorem 4. 1, (4. 9) converges uniformly on $GF[[q, x]]$.

As the uniform limit of a sequence of polynomial functions, the series (4. 9) represents some continuous function on $GF[[q, x]]$. That this function is in fact f follows from the observation that on $GF[q, x]$, which is dense in $GF[[q, x]]$, (4. 9) reduces to a finite series which obviously represents f. Finally, suppose that

$$f(t) = \sum_{i=0}^{\infty} A_i \frac{\Psi_i(t)}{F_i}. \tag{4. 12}$$

Forming the polynomial partial sums,

$$f_n(t) = \sum_{i=0}^{n} A_i \frac{\Psi_i(t)}{F_i}, \tag{4. 13}$$

of (4. 12), we have, by (2. 6) and (2. 7),

$$A_i = \Delta^i f_n(1) = \Delta^i f(1), \tag{4. 14}$$

since $f(t)$ and $f_n(t)$ agree on the set $\{1, x, \ldots, x^n\}$. Hence the representation (4. 9) is unique.

5. Differentiable linear operators

Given a continuous linear operator f on the $GF(q)$-vector space $GF[[q, x]]$, it is clear that the differentiability of f at 0 is equivalent to its differentiability everywhere in $GF[[q, x]]$. Suppose that f is represented by the interpolation series

$$f(t) = \sum_{i=0}^{\infty} A_i \frac{\Psi_i(t)}{F_i}, \tag{5. 1}$$

where $\{A_i\}$ is the null sequence $\{\Delta^i f(1)\}$. Then, by (2. 4), (2. 12), and (2. 13), the difference quotient at 0, denoted $D(t)$, is given by

$$D(t) = \frac{f(t)}{t} = \sum_{i=0}^{\infty} A_i \frac{\Psi_i(t)}{tF_i} = \sum_{i=0}^{\infty} \frac{A_i}{L_i} \frac{G^*_{q^i-1}(t)}{g_{q^i-1}} \qquad (t \neq 0). \tag{5. 2}$$

Now if $\{A_i/L_i\}$ is a null sequence, then, as $|G^*_{q^i-1}(t)/g_{q^i-1}| \leq 1$, the right-most series in (5. 2) converges uniformly for *all* $t \in GF[[q, x]]$. As the uniform limit of polynomial functions this series represents a function continuous at every point of $GF[[q, x]]$. Hence $f'(0)$ exists and

$$f'(0) = \lim_{t \to 0} D(t) = \sum_{i=0}^{\infty} \frac{A_i}{L_i} \frac{G^*_{q^i-1}(0)}{g_{q^i-1}}. \tag{5. 3}$$

From (2. 2), (2. 3), and (2. 12) it follows that

$$G^*_{q^i-1}(0) = (-1)^i \frac{F_i}{L_i}. \tag{5. 4}$$

Hence,

$$f'(0) = \sum_{i=0}^{\infty} (-1)^i \frac{A_i}{L_i}. \tag{5. 5}$$

Thus we have proved

Theorem 5. 1. *Let f, a continuous linear operator on the $GF(q)$-vector space $GF[[q, x]]$, be given by*

$$f(t) = \sum_{i=0}^{\infty} A_i \frac{\Psi_i(t)}{F_i}. \tag{5. 6}$$

Then if $\{A_i/L_i\}$ is a null sequence, f is differentiable everywhere in $GF[[q, x]]$ with derivative

$$f'(t) = \sum_{i=0}^{\infty} (-1)^i \frac{A_i}{L_i}. \tag{5. 7}$$

In order to prove the converse of Theorem 5. 1, we require the following lemma:

Lemma 5. 1. *For all positive integers r and for all integers i such that $0 \leqq i \leqq r$,*

$$\frac{G^*_{q^i-1}(x^r)}{g_{q^i-1}} \equiv (-1)^i \pmod{x}. \tag{5. 8}$$

Proof. By (2. 2), (2. 3), (2. 4), (2. 12), and (2. 13),

$$\frac{G^*_{q^i-1}(x^r)}{g_{q^i-1}} = \frac{L_i}{F_i} \sum_{j=0}^{i} (-1)^{i-j} \begin{bmatrix} i \\ j \end{bmatrix} x^{(q^j-1)r} = (-1)^i + \sum_{j=1}^{i} (-1)^{i-j} \frac{L_i}{F_j L_{i-j}^{q^j}} x^{(q^j-1)r}. \tag{5. 9}$$

But each of the terms other than $(-1)^i$ in the above sum is congruent to 0 (mod x), for if $1 \leqq j \leqq i \leqq r$,

$$v\left(\frac{L_i}{F_j L_{i-j}^{q^j}} x^{(q^j-1)r}\right) = i + (q^j-1)r - q^j(i-j) - (1 + q + \cdots + q^{j-1}) \tag{5. 10}$$

$$= (r-i)(q^j-1) + jq^j - (1 + q + \cdots + q^{j-1}) > 0.$$

Theorem 5. 2. *Let f, a continuous linear operator on the $GF(q)$-vector space $GF[[q, x]]$, be given by (5. 6). If f is differentiable at 0, then $\{A_i/L_i\}_{i \geqq 0}$ is a null sequence.*

Proof. Suppose that $f'(0) = \lambda$. We show (I.) that $\{A_i/L_i\}$ must be bounded and (II.) that $\{A_i/L_i\}$ cannot be bounded but not null.

(I.) Suppose that $\{A_i/L_i\}$ is unbounded, so that $\limsup |A_i/L_i| = +\infty$. Then there exists a strictly increasing sequence of positive integers $\{i_r\}_{r \geqq 0}$ such that $\lim_{r\to\infty} |A_{i_r}/L_{i_r}| = +\infty$. By substituting an appropriate subsequence of i_r if necessary, we may assume, for $0 \leqq i < i_r$, that

$$\left|\frac{A_i}{L_i}\right| < \left|\frac{A_{i_r}}{L_{i_r}}\right|. \tag{5. 11}$$

Letting $t \to 0$ along x^{i_r}, we have, by (5. 2),

$$\lim_{r\to\infty} D(x^{i_r}) = \lim_{r\to\infty} \sum_{i=0}^{i_r} \frac{A_i}{L_i} \frac{G^*_{q^i-1}(x^{i_r})}{g_{q^i-1}} = \lambda. \tag{5. 12}$$

(By (1. 5) and (2. 12), $G^*_{q^i-1}(x^{i_r}) = 0$ if $i > i_r$). But,

$$\lim_{r\to\infty} \left| \sum_{i=0}^{i_r} \frac{A_i}{L_i} \frac{G^*_{q^i-1}(x^{i_r})}{g_{q^i-1}} \right| \tag{5. 13}$$

$$= \lim_{r\to\infty} \left| \sum_{i=0}^{i_r-1} \frac{A_i}{L_i} \frac{G^*_{q^i-1}(x^{i_r})}{g_{q^i-1}} + \frac{A_{i_r}}{x^{i_r}} \right|.$$

$$= \lim_{r\to\infty} \left| \frac{A_{i_r}}{L_{i_r}} \right| = +\infty,$$

where the last step follows from (5. 11) and the fact that $|x^{i_r}| = |L_{i_r}|$ and $|G^*_{q^i-1}(x^{i_r})/g_{q^i-1}| \leqq 1$. Thus we have derived a contradiction to (5. 12).

(II.) Suppose that

$$\lim_{t\to 0} \sum_{i=0}^{\infty} \frac{A_i}{L_i} \frac{G^*_{q^i-1}(t)}{g_{q^i-1}} = \lambda, \tag{5. 14}$$

and that $\{A_i/L_i\}$ is bounded but not null. As we may multiply the coefficients A_i by a fixed power of x and may further change finitely many of these coefficients arbitrarily without affecting the above assertion, there is no loss of generality in assuming that

$$(5.15)\qquad \left|\frac{A_i}{L_i}\right| \leqq 1 \text{ and} \lim_{i\to\infty} \sup \left|\frac{A_i}{L_i}\right| = 1.$$

As $|\,|$ is discrete, the second assertion of (5.15) is equivalent to the statement that $|A_i/L_i| = 1$ for an infinite number of indices i.

Letting $t \to 0$ in (5.14) along $\{x^r\}_{r\geqq 0}$, we have

$$(5.16)\qquad \lim_{r\to\infty} \sum_{i=0}^{r} \frac{A_i}{L_i} \frac{G^*_{q^i-1}(x^r)}{g_{q^i-1}} = \lambda.$$

In particular, there exists a positive integer s such that, for all $r \geqq s$

$$(5.17)\qquad \left| \sum_{i=0}^{r} \frac{A_i}{L_i} \frac{G^*_{q^i-1}(x^r)}{g_{q^i-1}} - \lambda \right| \leqq b.$$

Combining (5.17) with (5.8) we have, for all $n \geqq 0$,

$$(5.18)\qquad \sum_{i=0}^{s+n} (-1)^i \frac{A_i}{L_i} = \lambda + \sigma(n),$$

where $|\sigma(n)| \leqq b$. Hence for all $n \geqq 1$

$$(5.19)\qquad \left|(-i)^{s+n} \frac{A_{s+n}}{L_{s+n}}\right| = |\sigma(n) - \sigma(n-1)| \leqq b,$$

in contradiction to (5.15).

It follows from the preceding theorem that

$$(5.20)\qquad \sum_{i=0}^{\infty} x^i \frac{\Psi_i(t)}{F_i}$$

is a continuous nowhere differentiable linear operator on the $GF(q)$-vector space $GF[[q, x]]$.

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