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MANDATORY HIV TESTING, DRUG-SNIFFING DOGS, AND THE PROBATIVE VALUE OF A 'COLD HIT' : THE GOSPEL ACCORDING TO THE REVEREND THOMAS BAYES

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QUESTION:

IF I TOSS A SINGLE FAIR DIE, WHAT ARE THE ODDS (in favor) OF THROWING A “1” ?

NOT 1 in 6 ! THAT'S THE CHANCE (or PROBABILITY) OF THROWING A "1."

THE ODDS IN FAVOR OF THROWING A "1" ARE 1 to 5 (often written 1:5).

WHAT'S THE DIFFERENCE?

BOTH CHANCES AND ODDS ARE RATIOS:

CHANCE = F/T

(F "in" T), where

F = # of favorable cases

T = # of possible cases

ODDS = F:U (or F/U)

(F "to" U), where

U = # of unfavorable cases

- Chances are numbers between 0 and 1 (or between 0 and 100 if we use percentages).
- Odds can take <u>any</u> non-negative value.

(e.g., the odds against throwing 10 heads in a row are 1023:1)

- When chances are very small, odds and chances are very close in magnitude. Example:

  There is a

  1 in 175,223,510

  chance of winning the jackpot in Powerball.

The odds in favor of winning the jackpot in Powerball are

1 to 175,223,509

…. so the odds against winning are

175,223,509 to 1.

►-2

Case Study 1: Should premarital testing for HIV be mandatory?

- *Sensitivity* of Elisa test = 99.9%. Among 1000 HIV-infected individuals, on average, 999 will register positive.
- *Specificity* of Elisa test = 99.9%. Among 1000 healthy individuals, on average, 999 will register negative.

So let's ask ….

How would mandatory premarital HIV testing work out in Knox County?

•*Prevalence* of HIV infection in Knox County = 1 per 1000.

• In Knox County, around 4000 marriage licenses issued per year => 8000 individuals tested.

If 8000 *randomly* chosen people were given the Elisa test,

	D	not D	
+	8	8	16
-	0	7984	7984
	8	7992	8000

$PV^{+} = P(D|+) = 50\%$

$PV^{-} = P(\text{not } D|-) = 100\%$

8 False Positives!

But 4000 engaged couples are not a random sample. More realistically, for them, probably, prevalence < 1 in 8,000.

	D	not D	
+	1	8	9
-	0	7991	7991
	1	7999	8000

$PV^{+} = 11\%$ $PV^{-} = 100\%$

Still 8 false positives! ►-3

BAYES' RULE (Rev. Thomas Bayes, 1702-1761)

New odds =

Old odds × Bayes factor

This formula tells us how to revise our old odds in the light of new evidence (e.g., + on the Elisa test, an alert by a drug-sniffing dog, a genetic match to blood found at a crime scene)

►-4

What is the Bayes Factor (denoted by B) when …….

I. Evidence = + on the Elisa test (for engaged couples).

B = Sensitivity/ 100-specificity

	D	not D	
+	1	8	9
-	0	7991	7991
	1	7999	8000

(I) Old odds =1: 7999

(II) Bayes' factor =

99.9/ 0.1 = 999

(III) New odds = 999 : 7999 ≈ 1:8

II. Evidence = a drug-sniffing dog's alert.

B = P(alert, if drugs present)/

P(alert, if drugs absent)

- The Falco case (News-Sentinel, May-July, 2001, Laura Ayo and Randy Kenner). ►-5

How should a dog's "accuracy" be measured?

Jordan (in the Falco case): by the new odds of the presence of a drug, given an alert = 355: 645 (35.5% “batting average”), based on performance in the field (the analogue of PV^+)

Jarvis (in a case of Taz, Blec, and Shadow): by the Bayes factor =

90 % / % of alerts when

drugs absent,

based on a controlled test.

The Bayes factor remains stable across all environments. The posterior odds (like PV^+) vary with the prevalence of drugs in vehicles stopped by the police.

Which measure is most relevant to the issue of whether a dog's alert constitutes probable cause for a search?

Let's conclude by looking at one more example….

III. Evidence = a match to genetic material found at a crime scene, and assumed to belong to the perpetrator.

B ≈ 100 / 100 – P(exclusion)

In the O.J. Simpson case, the probability of exclusion was

99.57%. (i.e., 0.43% of the population have the blood type found at the murder scene). So

the Bayes factor ≈ 233

If an individual has that blood type, his or her old odds of being guilty are multiplied by the factor 233 to get the new odds.

What are the old odds? If we select an inhabitant of the USA <u>at</u> <u>random</u>, test his or her blood, and get a match (a "cold hit"), the old odds are ≈ 1: 300 million. So the new odds are ≈ 233: 300 million.

What is the probative value of a "cold hit," the Bayes factor, or the new odds ? It depends. The new odds take account of all evidence, old as well as new. The Bayes factor measures the impact of the new evidence alone.

- The Bayesian juror:

Suppose that you are a juror who, having heard all but the serological evidence, regards the odds of O.J.'s guilt as 1:1.

Then finding out that O.J.'s blood type matches that found at the crime scene should increase your odds to 233:1 (your subjective estimate of the probability of guilt ≈ 99.6%)

I rest my case for reserving the term "odds" for genuine odds.

Recommended reading……

The Theory that Would Not Die: How Bayes' Rule Cracked the Enigma Code, Hunted Down Russian Submarines, and Emerged Triumphant from Two Centuries of Controversy, by Sharon Bertsch McGrayne

Yale University Press 2011

THANK YOU