

$$y'' - xy' = 0$$

$$\sum_{n=2}^{\infty} n(n-1) C_n x^{n-2} - x \sum_{n=1}^{\infty} n C_n x^{n-1} = 0$$

$$\sum_{n=2}^{\infty} n(n-1) C_n x^{n-2} - \sum_{n=1}^{\infty} n C_n x^n = 0$$

$$\sum_{n=0}^{\infty} (n+2)(n+1) C_{n+2} x^n - \sum_{n=1}^{\infty} n C_n x^n = 0$$

$$2C_2 + \sum_{n=1}^{\infty} (n+1)(n+2) C_{n+2} x^n - \sum_{n=1}^{\infty} n C_n x^n = 0$$

$$2C_2 + \sum_{n=1}^{\infty} \left( (n+1)(n+2) C_{n+2} - n C_n \right) x^n = 0$$

$\therefore C_2 = 0$  then

$$C_{n+2} = \frac{n C_n}{(n+1)(n+2)}, \quad n \geq 1$$

$$y = C_1 \left( x + \frac{1}{3!} x^3 + \frac{3}{5!} x^5 + \frac{1 \cdot 3 \cdot 5}{7!} x^7 + \dots \right)$$