Picard's Method HW

$$\frac{dy}{dx} = 2x + y \quad , \quad y(1) = 1 \leftarrow y_0$$

$$\phi_1(x) = 1 + \int_1^x (2t + 1)\, dt$$

$$= 1 + (t^2 + t)\Big|_1^x = 1 + \left[(x^2 + x) - (1+1)\right]$$

$$= 1 + x^2 + x - 2$$

$$\boxed{\phi_1(x) = x^2 + x - 1}$$

$$\phi_2(x) = 1 + \int_1^x \left(2t + (t^2 + t - 1)\right) dt$$

$$= 1 + \int_1^x \left(t^2 + 3t - 1\right) dt = 1 + \left(\tfrac{t^3}{3} + \tfrac{3}{2}t^2 - t\right)\Big|_1^x$$

$$\phi_2(x) = 1 + \left[\left(\tfrac{x^3}{3} + \tfrac{3}{2}x^2 - x\right) - \left(\tfrac{1}{3} + \tfrac{3}{2} - 1\right)\right]$$

$$\phi_2(x) = 1 + \tfrac{x^3}{3} + \tfrac{3}{2}x^2 - x - \tfrac{5}{6}$$

$$\phi_2(x) = \tfrac{x^3}{3} + \tfrac{3}{2}x^2 - x + \tfrac{1}{6}$$

$$\phi_3(x) = 1 + \int_1^x \left(2t + \left(\frac{t^3}{3} + \frac{3}{2}t^2 - t + \frac{1}{6}\right)\right) dt$$

$$= 1 + \int_1^x \left(\frac{t^3}{3} + \frac{7}{3}t^2 - t + \frac{1}{6}\right) dt$$

$$= 1 + \left(\frac{t^4}{12} + \frac{7}{9}t^3 - \frac{1}{2}t^2 + \frac{1}{6}t\right)\Big|_1^x$$

$$= 1 + \left[\left(\frac{x^4}{12} + \frac{7}{9}x^3 - \frac{1}{2}x^2 + \frac{1}{6}x\right) - \left(\frac{1}{12} + \frac{7}{9} - \frac{1}{2} + \frac{1}{6}\right)\right]$$

$$= 1 + \frac{x^4}{12} + \frac{7}{9}x^3 - \frac{1}{2}x^2 + \frac{1}{6}x - \frac{19}{36}$$

$$\phi_3(x) = \frac{x^4}{12} + \frac{7}{9}x^3 - \frac{1}{2}x^2 + \frac{1}{6}x + \frac{17}{36}$$